$$\frac{\sqrt[3]{729x^3}}{\sqrt[3]{27x^5}}$$

$$9\overline{)729}\quad\overset{81}{}$$

$$\begin{array}{r} 81 \\ 9\overline{)729} \end{array}$$

$729 = 9 \cdot 81 = 9 \cdot 9^2 = 9^3$

$27 = 9 \cdot 3 = 3^2 \cdot 3 = 3^3$

$$\frac{\sqrt[3]{729x^3}}{\sqrt[3]{27x^5}} = \frac{\sqrt[3]{729}\,\sqrt[3]{x^3}}{\sqrt[3]{27}\,\sqrt[3]{x^5}} = \frac{9x}{3x^{5/3}} = 3x^{1-5/3} = 3x^{3/3-5/3}$$

$$= 3x^{-2/3}$$

$$= \frac{3}{x^{2/3}}$$

$$= \frac{3}{\sqrt[3]{x^2}}$$

① $\dfrac{9}{x} - \dfrac{18}{x+9} - \dfrac{x}{(x+9)^2} = \dfrac{?}{x(x+9)^2}$

$$\begin{array}{r} 81 \\ 2 \\ \hline 162 \end{array}$$

Common denominator: $x(x+9)^2$

③ $\dfrac{(x+9)^2}{(x+9)^2}\left(\dfrac{9}{x}\right) - \dfrac{x(x+9)}{x(x+9)}\left(\dfrac{18}{x+9}\right) - \dfrac{x}{x}\left(\dfrac{x}{(x+9)^2}\right)$ ①=②

④ $$\frac{9(x+9)^2 - x(x+9)\cdot 18 - x^2}{x(x+9)^2}$$

$$\frac{9(x^2+2\cdot 9\cdot x+9^2) - 18(x^2+9x) - x^2}{} = \frac{9x^2+162x+729-18x^2-162x-x^2}{}$$

$$\frac{9x^2+162x+729-18x^2-162x-x^2}{x(x+9)^2}$$

(2)

$$\frac{x^2(9-18-1)+729}{x(x+9)^2} = \boxed{\frac{-10x^2+729}{x(x+9)^2}}$$

$ax^2+bx+c=0$ Quadratic Equation

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$(x+a)^2 = (x+a)(x+a) = x^2+ax+ax+a^2$

$= x^2+2ax+a^2$

$(x+9)^2 = (x+9)(x+9) = x^2+9x+9x+9^2$

$= x^2+2\cdot 9\cdot x+9^2$

$(x+a)(x-a) = x^2-a^2$

Complete The Square

Given x^2+bx, add $\left(\frac{b}{2}\right)^2$ to get

$$x^2+bx+\left(\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)^2$$

$$x^2+bx+\left(\frac{b}{2}\right)^2 = x^2+2\left(\frac{b}{2}\right)x+\left(\frac{b}{2}\right)^2$$
$$=\left(x+\frac{b}{2}\right)^2$$

E.g.: $x^2+7x+\left(\frac{7}{2}\right)^2=\left(x+\frac{7}{2}\right)^2$

Check: $\left(x+\frac{7}{2}\right)\left(x+\frac{7}{2}\right) = x^2+\frac{7}{2}x+\frac{7}{2}x+\left(\frac{7}{2}\right)^2$
$$=x^2+7x+\left(\frac{7}{2}\right)^2 \checkmark$$

E.g.: Solve $x^2+2x-5=0$ by completing the square

$x^2+2x+1-5=1$

Factor ↳ $(x+1)^2-5=1$

$(x+1)^2=6$

$x+1=\pm\sqrt{6}$

$x=-1\pm\sqrt{6}$

Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-2\pm\sqrt{2^2-4(1)(-5)}}{2}$$
$$=\frac{-2\pm\sqrt{4+20}}{2}$$
$$=\frac{-2\pm\sqrt{24}}{2}$$
$$=\frac{-2\pm\sqrt{4\cdot6}}{2}=\frac{-2\pm\sqrt{4}\sqrt{6}}{2}$$
$$=\frac{-2\pm2\sqrt{6}}{2}=\frac{2(-1\pm\sqrt{6})}{2}=-1\pm\sqrt{6}$$

Check

$x = -1 \pm \sqrt{6}$ $\quad x^2 + 2x - 5 = 0$

$(-1+\sqrt{6})^2 + 2(-1+\sqrt{6}) - 5 = (-1)^2 + 2(-1)\sqrt{6} + (\sqrt{6})^2$
$+ (-2 + 2\sqrt{6}) - 5$

$= +1 - 2\sqrt{6} + 6 - 2 + 2\sqrt{6} - 5$
$= 7 - 7$
$= 0$

$(-1-\sqrt{6})^2 + 2(-1-\sqrt{6}) - 5$

$= (-1)^2 + 2(-1)(-\sqrt{6}) + (\sqrt{6})^2 - 2 - 2\sqrt{6} - 5$
$= 1 + 2\sqrt{6} + 6 - 2 - 2\sqrt{6} - 5$
$= 7 - 7$
$= 0.$