Domain of A function

The domain of a function $f(x)$ the set of all values x such that $f(x)$ is defined.

Eg: $f(x) = x - 5$

$f(2) = 2 - 5 = -3$.

$f(8) = 8 - 5 = 3$

The domain of f is all real numbers, $\mathbb{R}$ or $(-\infty, \infty)$, $\{x \mid x \text{ is a real number}\}$.

Eg: $f(x) = \frac{1}{x-1}$

$f(5) = \frac{1}{5-1} = \frac{1}{4}$.

Domain: all real numbers except $x = 1$

$(-\infty, 1) \cup (1, \infty)$

$\{x \mid x \text{ is a real number and } x \neq 1\}$.

Eg: $g(x) = \sqrt{x-1}$

Since we can only take the square root of non-negative numbers (and get a real number), so we must have

$0 \le x - 1 \Rightarrow 1 \le x$, $[1, \infty)$

Eg: (Net Change of a function):

An astronaut weighs 130 lbs on earth. Her weight when she is h miles above the earth is given by

$$w(h) = 130\left(\frac{3960}{3960+h}\right)^2$$

Find the net change in weight from a height of 100 miles above the earth to a height of 400 miles above the earth.

$$w(400) - w(100) = 130\left(\frac{3960}{3960+400}\right)^2 - 130\left(\frac{3960}{3960+100}\right)^2$$

Using a calculator

$$\left.\begin{aligned} w(400) &= 107 \\ w(100) &= 124 \end{aligned}\right\} \text{rounded (approximate)}$$

So the net change is

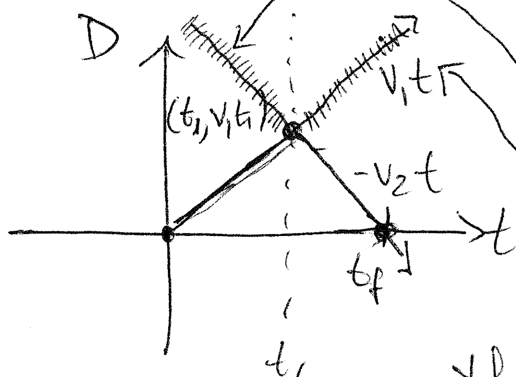
$$107 - 124 \approx -17$$

Piecewise Defined Functions

A piecewise defined function is a function defined by different rules on different parts of its domain.

Distance is a function of time: fixed velocity, $D = vt$, D is distance and t is time.

Two velocities, v_1 and v_2, a ball moves in a vacuum toward a wall at velocity v_1, hits the wall and returns at velocity v_2. Say the distance towards the wall is modeled by $D = v_1 t$ and the distance afterward is modeled by $D = -v_2 t$. Say the ball hits the wall at time t_1

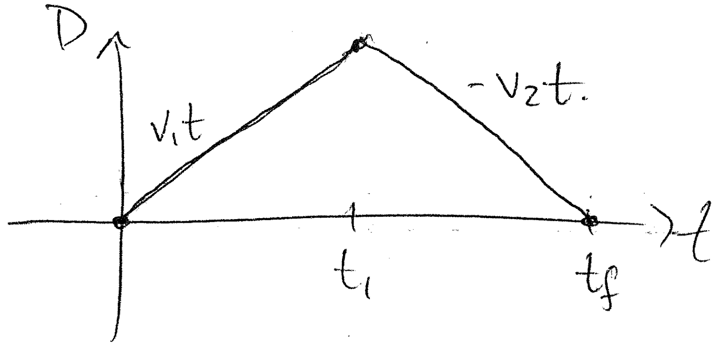


To make this a function, we want to delete these two line segments. This is a perfect place to define D as the piecewise function

$$D(t) = \begin{cases} v_1 t & [0, t_1] \\ -v_2 t & [t_1, t_f] \end{cases}$$

here, t_f is when the ball returns to its starting point.

The graph of $D(t)$ is



Eg: Cell phone plan

A cell phone plan has a basic charge of \$39/mo
The plan includes 400 minutes and charges 20¢ for each additional minute. If x represents minutes and c charge,

$$c(x) = \begin{cases} 39 & 0 \le x \le 400 \\ 39 + 0.20(x-400) & 400 \le x \end{cases}$$