Some Logic

A statement is a sentence that can be checked for truth, i.e. it's either true or false.

Implication

Implications have an if —, then — structure

If p and q are two logical statements, the symbols

$$P \Rightarrow q$$

which mean

"If p is true, then q is also true."

E.g.: "If you get straight As, then I will buy you a car."

p - "you get straight As."

q - "I buy you a car"

$P \Rightarrow q$

4 scenarios

(1) you get straight As, and I buy you a car.

(2) " " " " , " " don't buy you a car,

(3) you don't get straight As, I buy you a car.

(4) " " " " " , I don't buy you a car

p	q	p ⇒ q
T	T	T
T	F	F
F	T	T
F	F	T

E.g.: If no vertical line passes through more than one point on a graph, then this graph is the graph of a function.

Converse

Given an implication $p \Rightarrow q$, the converse of $p \Rightarrow q$ is $q \Rightarrow p$.

Logical Equivalence

Two valid implications $p \Rightarrow q$ and $q \Rightarrow p$. Write this $p \Leftrightarrow q$, read p if and only if q.

Eg.: Vertical Line Test.

This translates an algebraic condition to a geometric condition.

Warning: The converse, in general, is not always true in general.

Eg.: This a valid implication:

If it is raining, then there are clouds.

Converse:

If there are clouds, then it is raining.

This is not true, as there are cloudy days where it is not raining.

E.g.: Vacuous Truth

$1 = 2$

Pf: Let $a = b$.

$\Rightarrow a^2 = ab$ (mult. both sides by a)

$\Rightarrow a^2 + a^2 = 2a^2 = a^2 + ab$ (add a^2 to both sides)

$\Rightarrow 2a^2 - 2ab = a^2 + ab - 2ab = a^2 - ab$ (subtract $2ab$ from both sides)

$\Rightarrow 2(a^2 - ab) = 1 \cdot (a^2 - ab)$ (factor)

$\Rightarrow 2 = 1$ (divide by $a^2 - ab$) ▨ oops! Division by zero.

Eg: $5z + w^2 = 8$

a) Does this equation define z as a function of w?

$5z + w^2 = 8$

$\Rightarrow 5z = 8 - w^2$ (subtract w^2 from both sides)

$\Rightarrow z = \frac{8 - w^2}{5}$ (divide both sides by 5).

This shows that z is a function of w.

b) Does this equation define w as a function of z?

$5z + w^2 = 8$

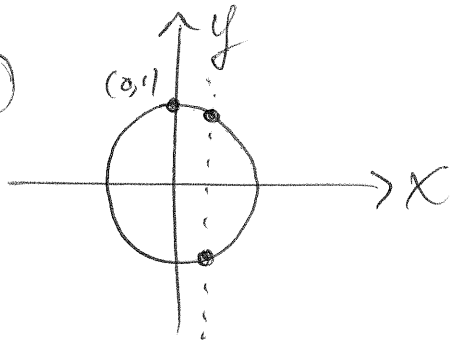
$\Rightarrow w^2 = 8 - 5z$ (subtract $5z$ from both sides).

The solutions to this equation are

$w = \sqrt{8-5z}$ and $w = -\sqrt{8-5z}$.

Given a value of z, there is a <u>choice</u> of value for w: either $w = \sqrt{8-5z}$ or $w = -\sqrt{8-5z}$. So, this equation does <u>not</u> define w as a function of z.

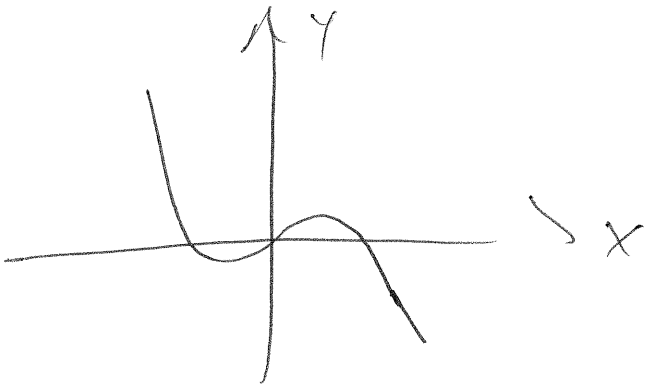
Eg: Decide whether ~~this~~ the graph determines y as a function of x.

① 

this vertical line intersects the graph in two points, so no.

$x^2+y^2=1$.

② $y = 3x - x^3$



Function: passes the vertical line test.

1.5 Function Notation

A function f is a rule that assigns to each input exactly one output. If we write x for the input and y for the output, then we use the notation $y = f(x)$ to describe f.

Eg: $y = x + 2$.

Express this as $y = f(x)$, where $f(x) = x + 2$ is the rule "add 2 to x."

Net Change:

The net change of a function f as x changes from a to b, $a \leq b$, is

$$f(b) - f(a).$$