Direct Proportionality

We say that the variable y is directly proportional to the variable x (or y varies directly with x) if x and y are related by an equation of the form $y = kx$, where k is called the constant of proportionality

Basically, this is a line with slope k passing through the origin, $(0,0)$.

Eg: 12 solar panels produce 2.4 kw of energy.

a) find the linear equation that ~~models~~ relates the number of panels installed to the kw of energy produced

Want: $y = kx$ on $[0, \infty)$

Know when y (representing energy) is 2.4 when x (representing the # of solar panels) is 12. So

$$2.4 = k\,12$$

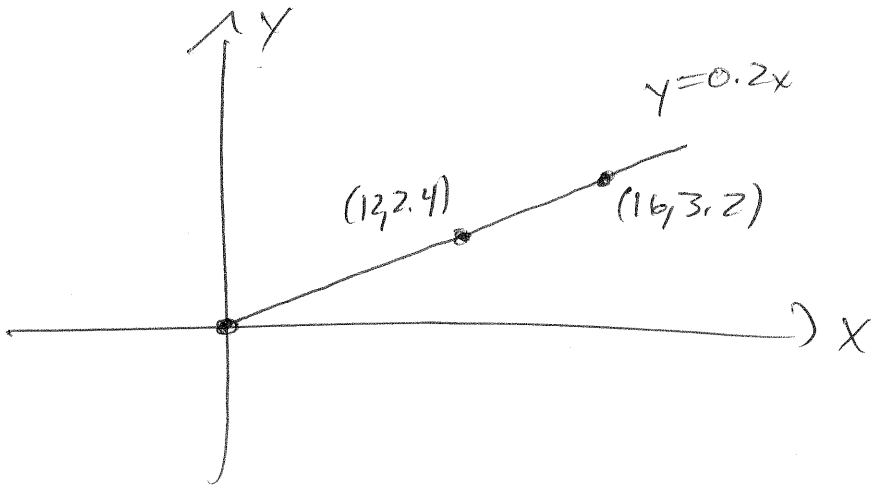
$$\Rightarrow \frac{2.4}{12} = k$$

$$\Rightarrow k = 0.2.$$

b) How many kw are produced by 16 panels? (2)

$$y = 0.2(16) = 3.2 \text{ kw}$$

c) Sketch a graph of $y = 0.2x$.



2.6: Linear Equations: Getting Information from a Model

Read this Section

E.g: "naturally flavored" soda, contains 5% o.j.
A new regulation stipulates ~~anything~~ any soda "natural" must contain at least 10% juice.
Manufacturer has a 900 G vat and adds o.j. directly.

a) Construct a model that gives the fraction of the mixture that is pure o.j. (3)

Let x be the amount of orange juice that is added to the vat.

Given: the vat contains 900 gallons of soda

5% of this is o.j.

So this means the amount of o.j in the vat is given by

$$900 \cdot \frac{5}{100} = 45 \text{ gallons of o.j.}$$

Total liquid content in the vat after adding x gallons of o.j. is

$$900 + x.$$

The total o.j. content is after adding x gallons of o.j. is

$$45 + x$$

So, the percentage of orange juice content after adding x gallons of o.j. is

$$\frac{\text{gallons of o.j.}}{\text{total liquid content}} = \frac{45 + x}{900 + x}$$

b) How much o.j must be added to satisfy the new rule? (11)

% o.j. content: $\frac{45+x}{900+x}$

Want to solve

$$\frac{1}{10} = \frac{10}{100} = \frac{45+x}{900+x}$$

for x. Mult. both sides by $900+x$

$$\Rightarrow \frac{900+x}{10} = 45+x \quad \text{(mult both sides by 10)}$$

$$\Rightarrow 900+x = 10(45+x) = 450+10x$$

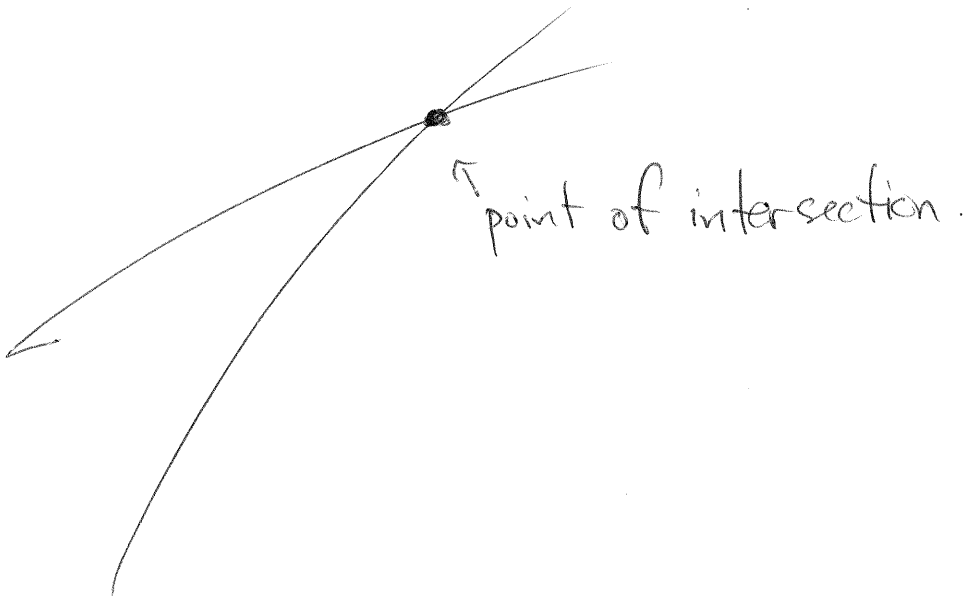
$$\Rightarrow 900-450 = 9x$$

$$\Rightarrow 450 = 9x$$

$$\Rightarrow x = \frac{450}{9} = \frac{45\cdot 10}{9} = \frac{9\cdot 5\cdot 10}{9} = 50.$$

So, adding 50 gallons of o.j. ensures that the soda has exactly 10% orange juice content.

Draw two non-parallel lines at random, they always intersect at a point!



These two lines have the equation

$$y = m_1 x + b_1 \quad \text{and} \quad y = m_2 x + b_2$$

If the lines intersect at a point (x_1, y_1), then we know that the point (x_1, y_1) lies on both lines, i.e.

$$y_1 = m_1 x_1 + b_1 \quad \text{and} \quad y_1 = m_2 x_1 + b_2$$

In particular, we can equate the expressions

$$m_1 x + b_1 = m_2 x + b_2$$

Solving this equation for x gives the value x_1 described above.

$$m_1x + b_1 = m_2x + b_2$$

$$\Rightarrow m_1x - m_2x = b_2 - b_1$$

$$\Rightarrow (m_1 - m_2)x = b_2 - b_1$$

Because these two lines are <u>not</u> parallel, this means $m_1 - m_2 \neq 0$, so we can divide both sides by $m_1 - m_2$ to get

$$x = \frac{b_2 - b_1}{m_1 - m_2}.$$

Eg: $y = 5x - 8$, $y = 3x + 2$. Where do they intersect?

Equate $5x - 8 = 3x + 2$ and solve for x.

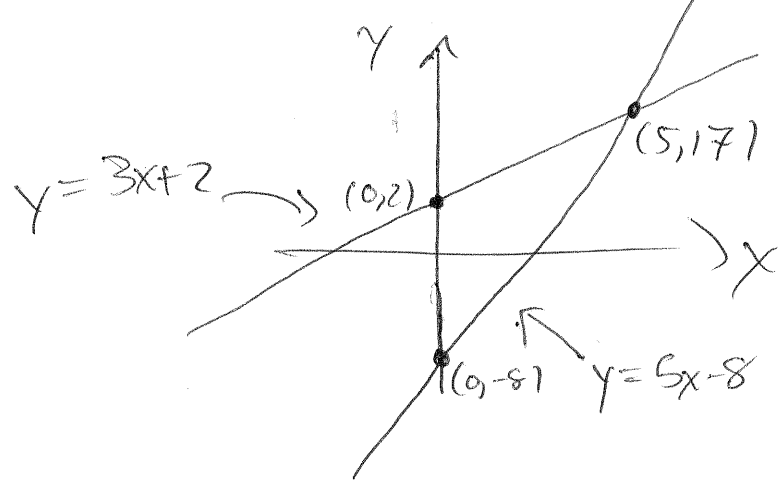
$$\Rightarrow 5x - 3x = 2 + 8$$

$$\Rightarrow 2x = 10$$

$$\Rightarrow x = 5.$$

$y = 5(5) - 8 = 25 - 8 = 17$ and $y = 3(5) + 2 = 15 + 2 = 17$.

So these two lines intersect at $(5, 17)$.



$y = m_1 x + b_1$, $y = m_2 x + b_2$, $m_1 \neq m_2$

$m_1 x + b_1 = m_2 x + b_2$

$\Rightarrow m_1 x - m_2 x + b_1 - b_2 = 0$

$\Rightarrow \underbrace{(m_1 - m_2)}_{m} x + \underbrace{(b_1 - b_2)}_{b} = 0.$

$mx + b = 0.$