Slope - Intercept Form

$y = mx + b$, m and b are constants.

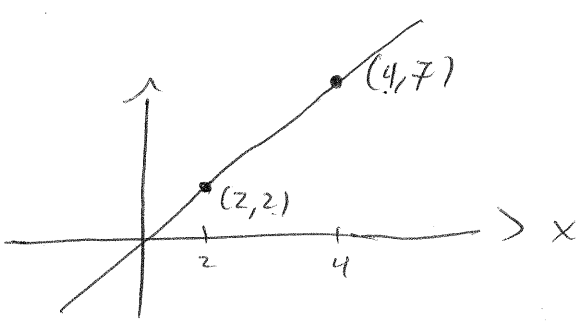
m is the slope or average rate of change (these words are synonymous in the case of lines)

b is the y-coordinate of the y-intercept, $(0,b)$.

E.g.: ① Find the equation of the line with slope 3 and y-intercept -2.

$f(x) = 3x - 2$.

② Find the line passing through the points $(2,2)$ and $(4,7)$



Find the slope of this line

$$m = \frac{7-2}{4-2} = 5/2$$

Want a function of the form $y = mx + b = 5/2 x + b$

Use the point $(2,2)$ to solve for b.

$$y = \tfrac{5}{2}x + b$$

$$2 = \left(\tfrac{5}{2}\right)(2) + b$$

$$\Rightarrow \ 2 = 5 + b$$

$$\Rightarrow \ 2 - 5 = -3 = b$$

So we have a linear function,

$$f(x) = \tfrac{5}{2}x - 3.$$

We know $f(2) = \tfrac{5}{2}(2) - 3 = 5 - 3 = 2$, so $f(x)$ passes through $(2,2)$. Check that it passes through $(4,7)$:

$$f(4) = \tfrac{5}{2}(4) - 3 = 5 \cdot 2 - 3 = 7.$$

So, yes. This is the line passing through $(2,2)$ and $(4,7)$.

Point-Slope Form

An equation of the line that passes through the point (x_1, y_1) and has slope m is

$$y - y_1 = m(x - x_1).$$

Rmk: To get back to ~~point-slope~~ slope-intercept form.

$$y - y_1 = m(x - x_1) = mx - mx_1$$

$$\Rightarrow \quad y = mx - mx_1 + y_1$$

$$= mx + \underbrace{(y_1 - mx_1)}_{b}$$

Equivalently, given a ~~gen~~ line in slope-intercept form, $y = mx + b$, we can get back to point-slope as follows:

$$y = mx + b \Rightarrow y - \underset{y_1}{b} = m(x - \underset{x_1}{0})$$

because this is the equation of a line with slope m passing through the point $(0, b)$.

E.g: Want a line through $(2,2)$ and $(4,7)$. (Part II)

Found the slope of the line was $m = 5/2$.

Using point-slope form we have the line

$$y - 2 = 5/2(x - 2)$$

$$y-2 = 5/2(x-2)$$

$$\Rightarrow y-2 = 5/2x - 5/2(2)$$
$$= 5/2x - 5$$

$$\Rightarrow y = 5/2x - 3.$$

E.g.: Find the equation of the line passing through $(1,-3)$ with slope $-1/2$.

$$y-(-3) = \frac{-1}{2}(x-1) \quad \text{(point-slope)}$$

$$\Rightarrow y+3 = \frac{-1}{2}x + \frac{1}{2}$$

$$\Rightarrow y = -\frac{1}{2}x + \frac{1}{2} - 3$$
$$= -\frac{1}{2}x + \frac{1}{2} - \frac{6}{2}$$
$$= -\frac{1}{2}x - 5/2. \quad \text{(slope-intercept)},$$

General Form

The general form of a line is

$$Ax + By + C = 0$$

where A and B are not both zero.

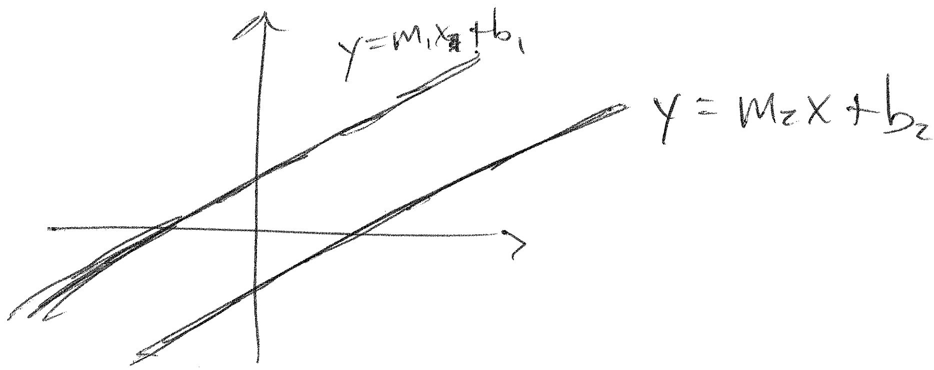
When $A=0$, $By+C=0$, so $y=-C/B$ is a horizontal line.

When $B=0$, $Ax+C=0$, so $x=-C/A$ is a vertical line

Parallel lines

Given two slopes m_1 and m_2, the lines $y = m_1 x + b_1$ and $y = m_2 x + b_2$ are parallel if and only if $m_1 = m_2$, $b_1 \neq b_2$.

Geometrically: E.g.:



E.g.: Finding parallel lines

Given the line $y = -3x + 4$.

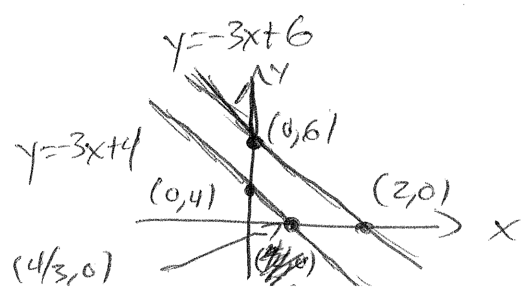
a) Find a line parallel to this one, passing through $(2,0)$

Given that the slope is $m = -3$, so the line is

$$y - 0 = -3(x - 2)$$

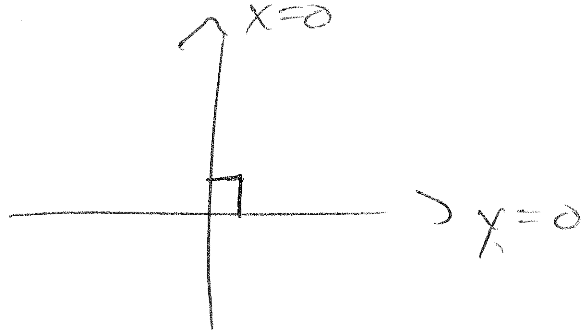
$$\Rightarrow y = -3x + 6.$$

b) Sketch.



Perpendicular Lines

Two lines are perpendicular if they meet at a right angle. E.g. $y=0$ and $x=0$ are perpendicular



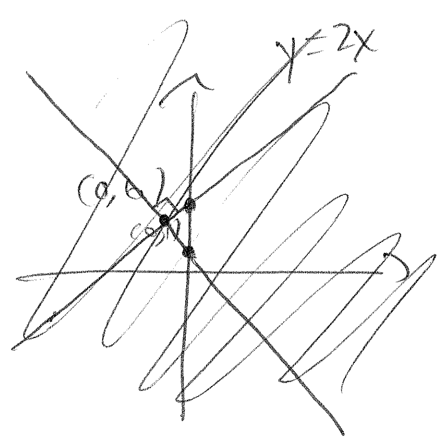
Suppose we have two non-vertical lines with slopes m_1 and m_2. Then these lines are perpendicular if and only if

$$m_1 = \frac{-1}{m_2} \quad (\text{or } \underline{m_1 m_2 = -1})$$

E.g.: The lines $y=x$ and $y=-x$ are perpendicular as $(-1)(1) = -1$.

E.g.: Given $y = 2x-6$, find a perp line passing through $(2,0)$.

Slope: $m = \frac{-1}{2}$



line:

$$y - 0 = -\tfrac{1}{2}(x-2)$$

$$\Rightarrow y = -\tfrac{1}{2}x + 1.$$

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