Eg:-

x hours	# tiles installed, $f(x)$
0	0
1	21
2	69
3	126
4	189
5	216
6	245
7	347
8	403

a) Find the average rate of installation in the first hour.

$$\frac{f(1)-f(0)}{1-0} = 21$$

b) Find the average rate of installation for the first four hours.

$$\frac{f(4)-f(0)}{4-0} = \frac{189}{4} = 47.25$$

c) Same for hours 4-6.

$$\frac{f(6)-f(4)}{6-4} = \frac{245-189}{2} = \frac{56}{2} = 28 \text{ tiles/hr}$$

$$\begin{array}{r} 245 \\ -189 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 28 \\ 2\overline{)56} \\ 4 \\ \hline 10 \end{array}$$

Average Speed of a Moving Object

For a moving object, let $s(t)$ be the distance it has travelled at a time t. Then the average rate of change of the function s from time t_1 to time t_2 is called the average speed,

$$\frac{f(t_2)-f(t_1)}{t_2-t_1}.$$

Eg.: Find the average rate of change for $f(x)=x^2+4$ between

a) $x=-3$ and $x=1$

$$\frac{f(-3)-f(1)}{(-3)-1} = \frac{(-3)^2+4-(1^2+4)}{-4}$$

$$= \frac{9+\cancel{4}-1-\cancel{4}}{-4}$$

$$= \frac{8}{-4}$$

$$= -2.$$

b) $x=2$ and $x=5$

$$\frac{f(2)-f(5)}{2-5} = \frac{(2^2+\cancel{4})-(5^2+\cancel{4})}{-3} = \frac{4-25}{-3} = \frac{-21}{-3} = 7.$$

Line

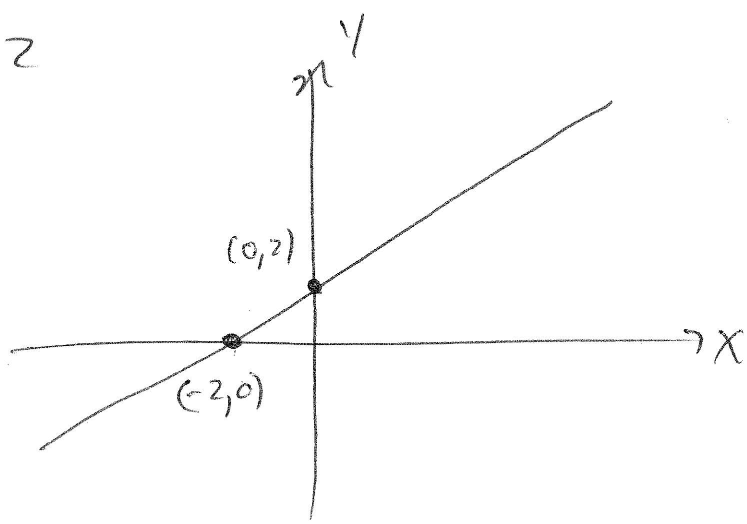
Slope-Intercept Form

$$f(x) = mx + b$$

is the equation of a line. Both m and b are constants, m is called the slope, and b is the y-coordinate of the y-intercept.

$f(0) = m \cdot 0 + b = b$, so the point $(0, b)$ lies on the graph of $f(x)$.

Ex: $f(x) = x + 2$



$x + 2 = 0$

$\Rightarrow x = -2$

The x-intercept is the point where the graph of $f(x)$ passes through the x-axis; know any point on the x-axis has y-coordinate 0, we obtain the x-coordinate by solving $f(x) = 0$.

Rmk: For a line $f(x) = mx + b$

i) The rate of change between any two points is constant (in fact, equal to m);

Between the points x_1 and x_2, we have

$$\frac{f(x_1) - f(x_2)}{x_1 - x_2} = \frac{(mx_1 + b) - (mx_2 + b)}{x_1 - x_2}$$

$$= \frac{mx_1 + \cancel{b} - mx_2 - \cancel{b}}{x_1 - x_2}$$

$$= \frac{m(x_1 - x_2)}{x_1 - x_2}$$

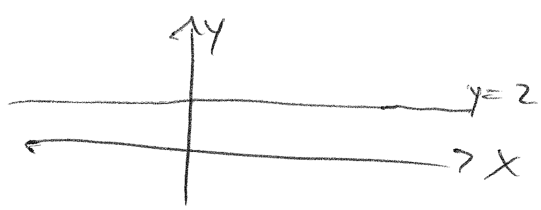
$$= m.$$

(ii) There are four possible ~~value~~ types of values for m:

a) $m = 0$. This is the horizontal line

$$f(x) = 0 \cdot x + b = b$$

E.g.: $f(x) = 2$



b) If $m > 0$, then $f(x) = mx + b$ is an increasing function.

If $x_1 < x_2$, then

$$\begin{aligned} f(x_2) - f(x_1) &= (mx_2 + b) - (mx_1 + b) \\ &= mx_2 + \cancel{b} - mx_1 - \cancel{b} \\ &= mx_2 - mx_1 \\ &= m(x_2 - x_1) > 0 \end{aligned}$$

since $x_1 < x_2 \Rightarrow 0 < x_2 - x_1$ and the product of two positive numbers is positive.

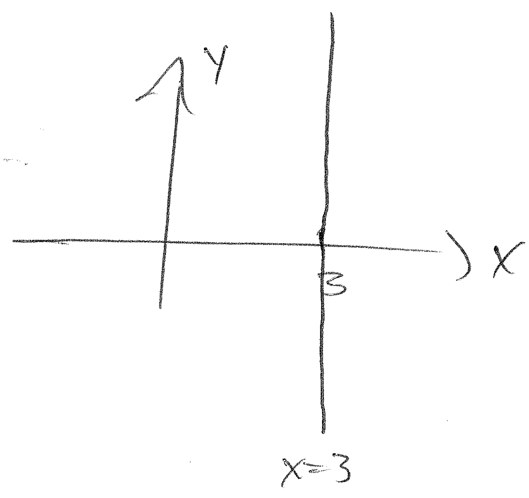
c) If $m < 0$, then $f(x) = mx + b$ is decreasing: $x_1 < x_2$

$$f(x_2) - f(x_1) = m(x_2 - x_1) < 0$$

Since $x_2 - x_1 > 0$ and $m < 0$.

d) The last class of lines are vertical lines; they have the form $x = b$

E.g.: $\underline{x = 3}$



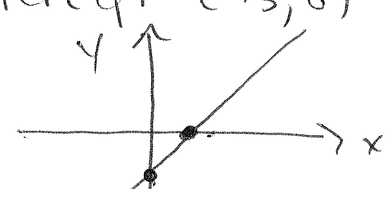
We say the slope is undefined.

E.g.: $f(x) = 3x - 2$ y-intercept: $(0, -2)$

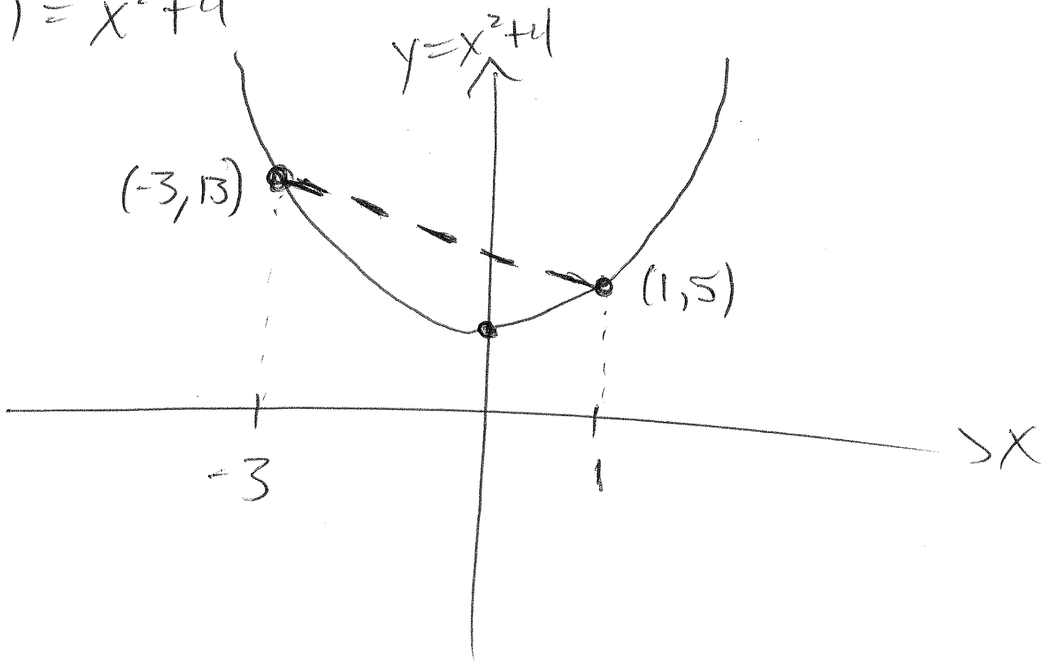
$0 = 3x - 2$ x-intercept: $(2/3, 0)$

$\Rightarrow 3x = 2$

$\Rightarrow x = 2/3$



$f(x) = x^2 + 4$



The average rate of change between $x = 3$ and $x = 1$ is the slope of the line connecting the points $(-3, 13)$ and $(1, 5)$.