d) For what depth is the volume greater than 60 in^3?

$V = h \cdot w \cdot d$

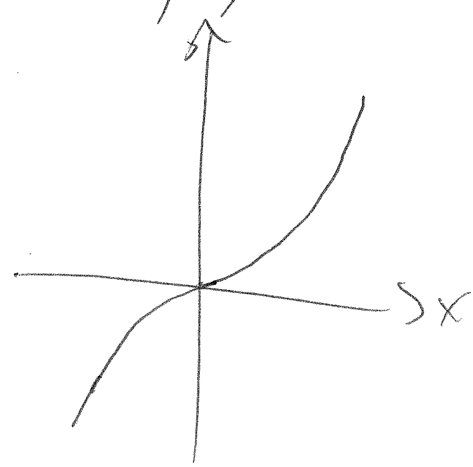
$h = 3d$

$w = 5d$

$V = (3d)(5d)d$

$= 15d^3$

$V(d) = 15d^3 > 60$



Solve $15d^3 = 60$

$\Rightarrow d^3 = 60/15 = 4$

$\Rightarrow d = \sqrt[3]{4}$

So if we take $d > \sqrt[3]{4}$, then

$V(d) = 15d^3 > 15(\sqrt[3]{4})^3 = 15 \cdot 4 = 60$

$d > \sqrt[3]{4} \approx 1.59$ in.

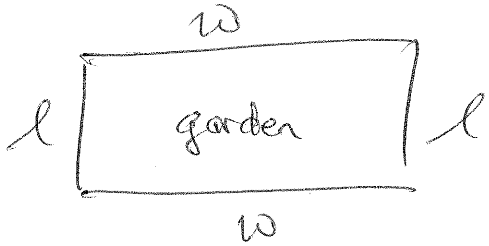
Rmk: By the graph on the right, $f(x) = x^3$ is an increasing function, so whenever $a < b$, $f(a) = a^3 < b^3 = f(b)$

$\sqrt[3]{4} < d, \Rightarrow f(\sqrt[3]{4}) = (\sqrt[3]{4})^3 = 4 < d^3 = f(d)$

$\Rightarrow 60 < 15d^3.$

Eg: A gardener has 140 ft of fencing to fence a rectangular garden.

a) Find a function that models the area of the garden she can fence.



$A(w, l) = wl$ (area)

$P(w, l) = 2l + 2w$ (perimeter)

$P(w, l) = 2l + 2w = 140$

$\Rightarrow 2l = 140 - 2w$

$\Rightarrow l = \dfrac{140 - 2w}{2} = \dfrac{140}{2} - \dfrac{2w}{2} = 70 - w$

$A = wl = w(70 - w) = 70w - w^2.$

$A(w) = 70w - w^2.$

b) For what range of widths is the area greater than $825\ ft^2$?

i.e. for what values of w is the inequality

$70w - w^2 > 825$

true?

Solve

$$70w - w^2 = 825$$

$$\Rightarrow \quad -w^2 + 70w - 825 = 0$$

$$w = \frac{-70 \pm \sqrt{(70)^2 - 4(-1)(-825)}}{2(-1)}$$

$$= \frac{-70 \pm \sqrt{4900 - 3300}}{-2}$$

$$= \frac{-70 \pm \sqrt{1600}}{-2}$$

$$= \frac{-70 \pm \sqrt{4^2 \cdot 10^2}}{-2}$$

$$= \frac{-70 \pm \sqrt{4^2} \cdot \sqrt{10^2}}{-2}$$

$$= \frac{-70 \pm 40}{-2}$$

$$w = \frac{-70 + 40}{-2} \quad \text{or} \quad w = \frac{-70 - 40}{2}$$

$$= 15 \qquad\qquad = 55$$

$$\begin{array}{l} 70 \\ \underline{70} \\ 4900 \end{array} \qquad \begin{array}{l} 825 \\ \underline{4} \\ 3300 \end{array} \qquad \begin{array}{l} 4900 \\ \underline{-3300} \\ 1600 \end{array}$$

$$1600 = 16 \cdot 100 = 4^2 \cdot 10^2$$

$$-(60)^2 + 70(60) - 825$$

$$-3600 + 4200 - 825$$

$$4200 - 4425 < 0$$

$$\begin{array}{l} 70 \\ \underline{20} \\ 1400 \end{array}$$

$$-(20)^2 + 70(20) - 825$$

$$-400 + 1400 - 825$$

$$1400 - 1225 > 0$$

$$\longleftarrow \overset{-}{} \underset{15}{|} \overset{+}{} \underset{55}{|} \overset{-}{} \longrightarrow$$

So, when $15 < w < 55$, Area is strictly greater than $825\,ft^2$.

c) Can the gardener fence an area of $1250\,ft^2$? ⑪

$$A(w) = 70w - \cancel{70}w^2$$

Is there a solution to

$$70w - w^2 = 1250\ ?$$

$$70w - w^2 = 1250$$

$$\Rightarrow \quad 70w = 1250 + w^2$$

$$\Rightarrow \quad w^2 - 70w + 1250 = 0$$

Recall

$ax^2 + bx + c = 0$, the discriminant is $b^2 - 4ac$.

$$(-70)^2 - 4(1)(1250) = 4900 - 5000 < 0.$$

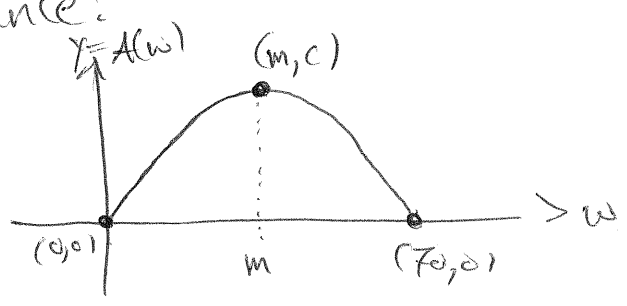
$$\begin{array}{r} \overset{1\ 2}{1250} \\ \times\ 4 \\ \hline 5000 \end{array}$$

So there is no solution to the equation

$$70w - w^2 = 1250$$

and the gardener cannot fence an area this large.

What is the largest possible area the gardener can fence?



Trace with a graphing calculator, this ~~value~~ point has width 35

$$A(35) = 70(35) - 35^2 = 1225.$$

Chapter 2: Linear Functions and Models

2.1: ~~Average~~ Average Rate of Change

Def^n: The average rate of change of the function $f(x)$ between $x=a$ and $x=b$ is

$$\cancel{f(x)} \quad \frac{f(b)-f(a)}{b-a} = \frac{f(a)-f(b)}{a-b}$$

$$\frac{f(b)-f(a)}{b-a} = \frac{-(f(a)-f(b))}{-(a-b)} = \frac{f(a)-f(b)}{a-b}.$$

Say you drive a distance of 75 miles in one hour. The average rate of change in distance is 75 mph. Note, this does not mean you drove 75 mph the entire time.