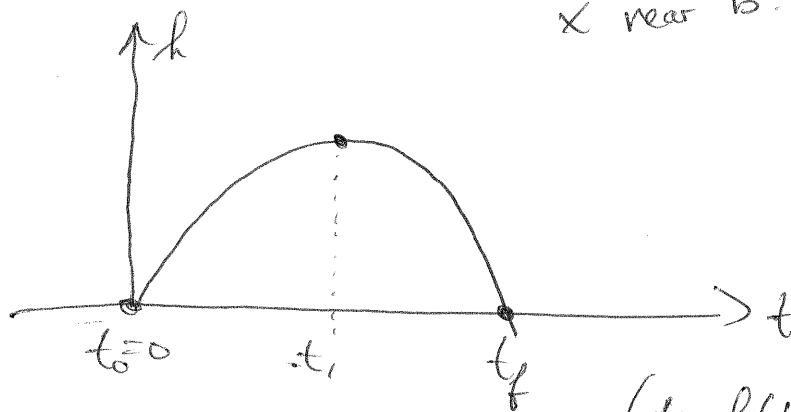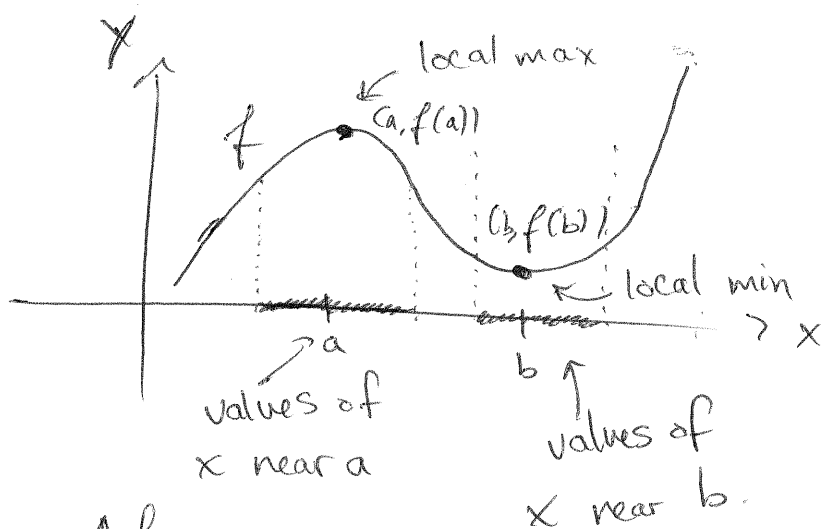# Local Maximum and Minimum Values

Def^n ① The function value $f(a)$ is a <u>local</u> <u>maximum</u> <u>value</u> of f if for values of x near a

$$f(x) \leq f(a).$$

② The function value $f(a)$ is a <u>local</u> <u>minimum</u> <u>value</u> of f if for values of x near a

$$f(a) \leq f(x).$$

If this graph describes the height of an object thrown upwards assuming only the force due to gravity, $(t, f(t))$ is the local max

Rmk: A local maximum occurs when the function changes from increasing to decreasing.

A local minimum occurs when the function changes from decreasing to increasing.

1.8 Working with functions: Modeling Real-World relationships

E.g.: A company manufactures baseball caps with school logos. The company charges a $500 fixed fee and $8 for each cap.

a) Find a linear model for the cost.

$$C(n) = 500 + 8n$$

b) How much will 225 caps cost?

$$C(225) = 500 + 8 \cdot 225$$
$$= \$2300.$$

$$\begin{array}{r} 2\,4 \\ 225 \\ \underline{8} \\ 1800 \\ 500 \\ \hline 2300 \end{array}$$

Eg. 3

- 1200 - gallon tank

Garden requires 80 gallons/day.

a) Find a function W that gives the amount of water in the tank x days after it has been filled.

$$W(x) = 1200 - 80x$$

b) Find out how much water is left after 3 days & 12 days

$$W(3) = 1200 - 80 \cdot 3$$
$$= 1200 - 240$$
$$= 960.$$

$$W(12) = 1200 - 80 \cdot 12$$
$$= 1200 - 960$$
$$= 240$$

$$\begin{array}{r} 80 \\ 12 \\ \hline 160 \\ 800 \\ \hline 960 \end{array}$$

c) Calculate W(20)

$$W(20) = 1200 - 80 \cdot 20$$
$$= -400$$

$$\begin{array}{r} 80 \\ 20 \\ \hline 1600 \end{array}$$

This tells us the tank ran dry between day 12 and day 20

d) How many days until the tank is empty?

The tank is empty when $\underline{W(x) = 0}$

$$\Rightarrow 1200 - 80x = 0$$

$$\Rightarrow 1200 = 80x$$

$$\Rightarrow \frac{1200}{80} = x$$

$$\Rightarrow x = \frac{120}{8} = 15.$$

$$\begin{array}{r} 15 \\ 8\overline{)120} \\ \underline{8} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

e) How many days until the tank has 200 gallons?

Solve: $W(x) = 200$ for x

$$\Rightarrow 1200 - 80x = 200$$

$$\Rightarrow 1000 - 80x = 0$$

$$\Rightarrow 1000 = 80x$$

$$\Rightarrow x = \frac{1000}{80} = \frac{100}{8}.$$

$$\begin{array}{r} 12 \\ 8\overline{)100} \\ \underline{8} \\ 20 \\ 16 \\ \underline{} \\ 4 \end{array}$$

$x = 12.5$ or the gardener must refill the tank every 12.5 days.

$100 = 8 \cdot 12 + 4/8$

$= 8 \cdot 12 + \frac{1}{2}$

Ex 4: Box that has width 3 times its depth
and height that is 5 times its depth.

a) Find a function that models the
volume of the box in terms of its
depth.

Volume of a box is

$\cancel{V = h \cdot w \cdot d}$, V - volume,
$V = h \cdot w \cdot d$ h - height,
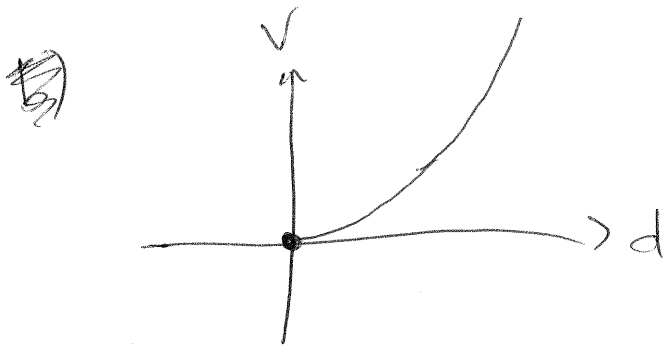 $\cancel{\text{length}}$ w - width
           d - depth

Given: $h = 5(d)$
       $w = 3(d)$

Get $V(d) = h(d) \cdot w(d) \cdot d = (5d)(3d)d = 15d^3$

b) Find the volume when $d = \frac{3}{2}$

$$V = 15\left(\frac{3}{2}\right)^3 = \frac{15 \cdot 27}{8} \approx 50.625 \, in^3$$

c) For what depth is the volume 90 in^3

$$V = 90 = 15d^3$$

$$\Rightarrow \frac{90}{15} = d^3$$

$$\Rightarrow 6 = d^3$$

$$\Rightarrow d = \sqrt[3]{6} \approx 1.82 \text{ in.}$$