

## 6.3 : Equations: Describing Relationships in Data (1)

Def<sup>n</sup>: A model is a mathematical representation (e.g. an equation) of a real world situation.

Modeling is the process of finding such a representation.

E.g.: Data

(ft)

Depth	Pressure (lb/in <sup>2</sup> )
0	14.7
10	19.2
20	23.7
30	28.2
200	?

Model

$$P = 14.7 + 0.45d$$

where  $P$  is pressure in lb/in<sup>2</sup> and  $d$  is depth in feet.

## Linear Models

A linear<sup>model</sup> is an equation of the form

$$y = A + Bx,$$

$A$  is the initial value, i.e. the value when  $x$  is 0, and  $B$  is the constant amount by which  $y$  increases for each unit increase in  $x$ .

Ex.: A furniture maker collects the following data

②

x chairs	c dollars
0	80
1	92
2	104
3	116
4	128

C is the cost of x  
number of chairs  
made.

a) Find a linear model for the cost C of making x chairs.

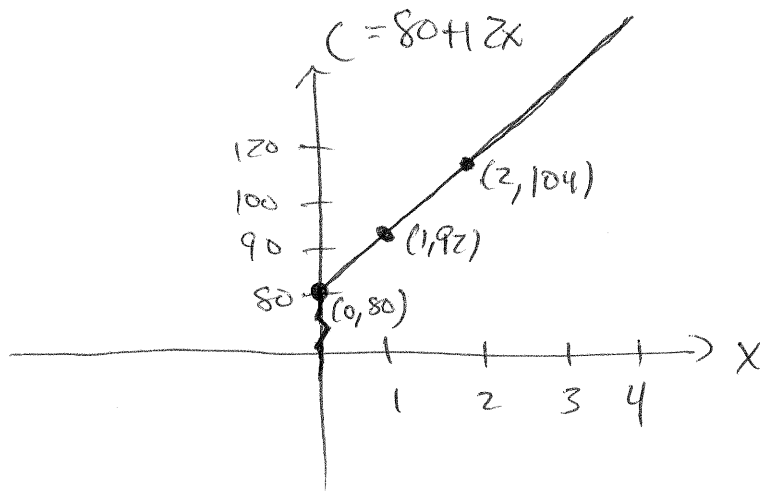
Recall:  $C = A + Bx$  where A is the initial value and B is the increase in C for a unit increase in x.

From the data, we see that the initial value is  $A = 80$ . From the table, we see that each unit increase in x causes an increase in C by 12, so  $B = 12$ . So our linear model is

$$C = 80 + 12x$$

b) Draw a graph of  $C = 80 + 12x$

③



### First Differences

For data with evenly spaced inputs,

- the first differences are the differences in successive outputs.
- If the first differences are constant, then there is a linear model for the data.

E.g.: A model for temperature and Elevation

Elevation (km)	Temp (°C)
0	20
1	10
2	0
3	-10
4	-20
5	-30

First differences are -10 (i.e. each increase in elevation by 1km decreases the temp. by 10°C.)

$$T = 20 - 10h$$

Depth (ft)	Pressure lb/in <sup>2</sup>
0	14.7
10	19.2
20	23.7
30	28.2
40	32.7
50	37.2

First Differences

$$19.2 - 14.7 = 4.5$$

$$23.7 - 19.2 = 4.5$$

$$28.2 - 23.7 = 4.5$$

$$32.7 - 28.2 = 4.5$$

$$37.2 - 32.7 = 4.5$$

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$$P = A + Bd$$

$$A = 14.7$$

To get a unit increase ~~in~~, we need to divide the first difference value by 10,

$$B = 4.5 / 10 = 0.45,$$

$$P = 14.7 + 0.45d.$$

## 1.4 Functions: Describing Change

Def<sup>n</sup>: A function is a relation in which each input gives exactly one output.

Fig:  $y = 2 + 3x$  is a function,  
any polynomial is also a function.

Eg.:  $A = \{1, 2, 3, 4\}$

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$B = \{5, 6, 7, 8\}$

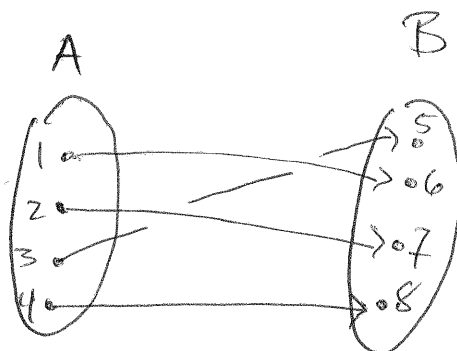
We can define a function  $f$  from  $A$  to  $B$  by the rules

$f(1) = 6$

$f(2) = 7$

$f(3) = 5$

$f(4) = 8$



A non-example:

$f(1) = 5$

$f(1) = 6$

$f(2) = 7$

$f(3) = 8$

} This fails to be a function because there is a choice for the output of  $f(1)$ .

