$$x^2 + 3x = 0$$

$$x^2 + 2\left(\frac{3}{2}\right)x = 0$$

$$x^2 + 2\left(\frac{3}{2}\right)x + \left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$$

$$x + \frac{3}{2} = \pm\sqrt{\left(\frac{3}{2}\right)^2} = \pm\frac{3}{2}$$

$$x = \pm 3/2 - \frac{3}{2}$$

$$x(x+3) = 0$$

So $x = 0$ or

$x = -3$.

$$x = \frac{3}{2} - \frac{3}{2} = 0 \quad \text{or} \quad x = -\frac{3}{2} - \frac{3}{2} = -\frac{6}{2} = -3.$$

---

<u>In general</u>

$$ax^2 + bx + c = 0$$

factor an a out of the leading 2 terms

$$a\left(x^2 + 2\left(\frac{b}{2a}\right)x\right) + c = 0$$

complete the square inside the parentheses

$$a\left(\underbrace{x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2}\, - \left(\frac{b}{2a}\right)^2\right) + c = 0$$

factor the perfect square

$$a\left(\left(x + \left(\frac{b}{2a}\right)\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c = 0$$

distribute the a

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$$

$$a\left(x+\frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c = \frac{b^2-4ac}{4a}$$

$$a\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a}$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

take the square root of both sides

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}} = \pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}} = \frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

(Quadratic Formula)

This says, the solutions to the polynomial equation $ax^2+bx+c=0$ are

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } x = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

If we have a degree 2 polynomial equation

$$ax^2+bx+c=0$$

there are three possibilities for the solutions and they are determined by the discriminant $D=b^2-4ac$.

Solutions: $x=\dfrac{-b\pm\sqrt{D}}{2a}$

If $D<0$ there are no real solutions.

E.g.: $x^2+9=0$

$$D=0^2-4(1)(9)=-36<0$$

No real solutions.

If $D=0$, there is ~~exa~~ exactly one real solution

$$x=\frac{-b\pm\sqrt{D}}{2a}=\frac{-b}{2a}.$$

E.g.: $x^2+6x+9$

$$D=6^2-4(1)(9)=36-36=0$$

Exactly one solution $x=\dfrac{-6}{2(1)}=-3$.

If $D>0$, two distinct solutions.

E.g.: $x^2 - 5x + 6 = 0$

$$D = (-5)^2 - 4(1)(6)$$
$$= 25 - 24$$
$$= 1.$$

$$x = \frac{-(-5) \pm \sqrt{1}}{2(1)}$$
$$= \frac{5 \pm 1}{2}$$
$$x = \frac{5+1}{2} = \frac{6}{2} = 3$$

and

$$x = \frac{5-1}{2} = \frac{4}{2} = 2$$

$$(x+2)(x+3) = x^2 + 5x + 6$$

$$D = 25 - 4(1)(6)$$
$$= 25 - 24 = 1.$$

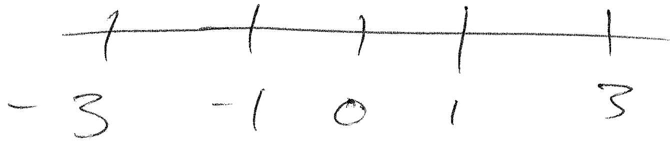
## C.3 Solving Inequalities

### Operations on Inequalities

1. Add/subtract the same quantity on both sides.
2. Multiply both sides by the same positive quantity.
3. Multiply both sides by the same negative quantity AND flip the inequality.

Eg. (3) $1 < 3$ and $-1 > -3$ or

$-3 < -1$.



## Linear Inequalities

All of the expressions are linear.

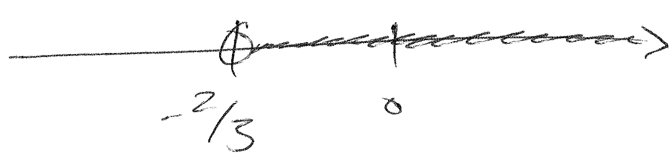
Ex.: $3x < 9x + 4$. $\rightarrow$ subtract $3x$ from both sides

$0 < 6x + 4$ $\rightarrow$ subtract 4 from both sides

$-4 < 6x$ $\rightarrow$ divide both sides by 6

$\frac{-4}{6} = \frac{-2}{3} < x$.

Pictorially, the solution set is



## Non-Linear

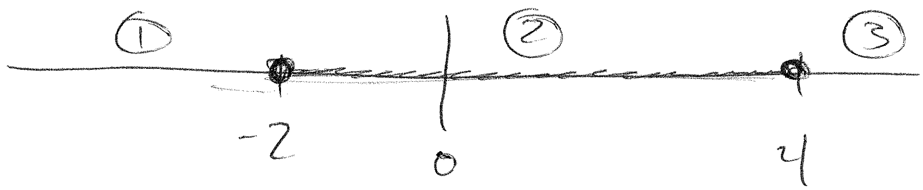
E.g.: $x^2 - 2x - 8 \le 0$.

$-4+2$
$= -2$

$(-4)(2) = -8$

$(x+2)(x-4) = x^2 - 2x - 8 \le 0$

When $x = -2$ or $x = 4$, both solutions to the inequality.




Check easy values in each of the three regions

1 $x < -2$ $x = -3$ ~~positive~~ positive

2 $-2 < x < 4$ $x = 0$ $0^2 - 2(0) - 8 = -8 < 0$ ✓

3 $4 < x$ $x = 5$ positive

Solutions: $-2 \le x \le 4$.