B.2 Factoring Algebraic Expressions

Factoring out Common Terms

Eg.: a) $3x^2 - 6x = (3)(x)(x) - (3)(2)(x)$

$= 3x(x-2)$

Check:

$3x(x-2) = 3x^2 - 6x$

b) $8x^4y^2 + 6x^3y^3 - 2xy^4 = 2xy^2(4x^3 + 3x^2y - y^2)$

c) Factor

$(2x+1)(x-3) - 5(x-3) = (x-3)(2x+1-5)$

$= (x-3)(2x-1)$

Factoring Trinomials

Eg.: Factor $x^2 + 7x + 12$

$(x+3)(x+4) = (x^2 + 4x + 3x + 12)$

$= x^2 + 7x + 12$

~~Factor~~

$(x+r)(x+s) = x^2 + sx + rx + rs$

$= x^2 + (s+r)x + rs$

Factor $6x^2+7x-5$

$(3x+5)(2x-1)$

$=$

$(3x)(2x) - 3x + 10x - 5$

$=$

$6x^2+7x-5$.

Special Formulas

If A, B are any real numbers

$A^2-B^2 = (A-B)(A+B)$ Difference of Two Squares

$A^2+2AB+B^2 = (A+B)^2$ } Perfect Squares

$A^2-2AB+B^2 = (A-B)^2$ }

Eg: a) $4x^2-25 = 2^2x^2 - 5^2$

$= (2x)^2 - 5^2$

$= (2x-5)(2x+5)$

b) $(x+y)^2 - z^2 = ((x+y)+z)((x+y)-z)$

$= (x+y+z)(x+y-z)$.

Recognizing Perfect Squares

$(A+B)^2 = A^2 + 2AB + B^2$

$(x+a)^2 = x^2 + 2ax + a^2$

Eg: a) x^2+6x+9

$x^2+6x+9 = x^2 + 2(3)x + 3^2$

$= (x+3)^2$.

b) $4x^2 - 4xy + y^2$ $\qquad (A-B)^2 = A^2 - 2AB + B^2$

$4x^2 - 4xy + y^2 = (2x)^2 - 2(2x)y + y^2$ ($A = 2x$, $B = y$)

$= (2x-y)^2$

Factoring By Grouping

Eg: Factor $x^3 + x^2 + 4x + 4$

$(x^3+x^2) + (4x+4) = x^2(x+1) + 4(x+1)$

$= (x+1)(x^2+4)$

Factor $x^3 - 2x^2 - 3x + 6$

$(x^3 - 2x^2) + (-3x+6) = x^2(x-2) - 3(x-2)$

$= (x-2)(x^2-3)$.

B.3 Rational Expressions

A rational expression is a ratio of two polynomials $f(x)$ and $g(x) \neq 0$, $\frac{f(x)}{g(x)}$.

E.g.: $\frac{2x}{x-1}$, $\frac{3}{x+2}$, $\frac{x^2-1}{x+1}$.

Simplifying Rational Expressions By Cancelling

E.g.: $\frac{x^2-1}{x^2+x-2} = \frac{(x+1)(x-1)}{(x-1)(x+2)} = \frac{x+1}{x+2}$.

Multiplying Rational Expressions:

Multiply the numerators & denominators.

E.g.: $\frac{x^2+2x-3}{x^2+8x+16} \cdot \frac{3x+12}{x-1}$

$\frac{(x+3)(x-1)}{(x+4)^2} \cdot \frac{3(x+4)}{(x-1)} = \frac{(x+3)\cdot 3}{x+4}$

Division

Given a rational expression $f(x)/g(x)$,

$$\frac{1}{\left(\frac{f(x)}{g(x)}\right)} = \frac{g(x)}{f(x)}, \quad f(x) \neq 0.$$

Eg:

$$\frac{x-4}{x^2-4} \div \frac{x^2-3x-4}{x^2+5x+6} = \frac{x-4}{x^2-4} \cdot \frac{x^2+5x+6}{x^2-3x-4}$$

$$= \frac{\cancel{x-4}}{\cancel{(x+2)}(x-2)} \cdot \frac{(x+3)\cancel{(x+2)}}{\cancel{(x-4)}(x+1)}$$

$$= \frac{x+3}{(x-2)(x+1)}$$