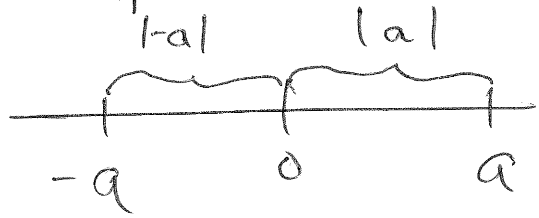
Recall: Absolute value of a number a is the distance from a to 0

Algebraically:

$$|a| = \begin{cases} a & \text{if } a \geq 0, \\ -a & \text{if } a < 0. \end{cases}$$

Geometrically: a is positive



Distance

If a and b are real numbers, then the distance from a to b is

$$d(a,b) = |a-b| = |b-a|$$

Rmk: One of three possibilities

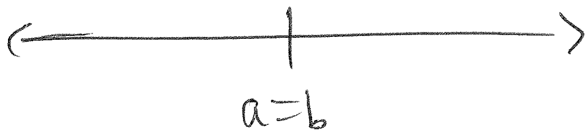
(i) $a = b$, so $a-b=0$

(ii) $a<b$, so $0<b-a$

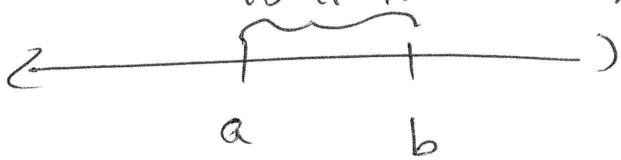
(iii) $b<a$, so $b-a<0$

$|a-b| = |-(b-a)| = |b-a|$

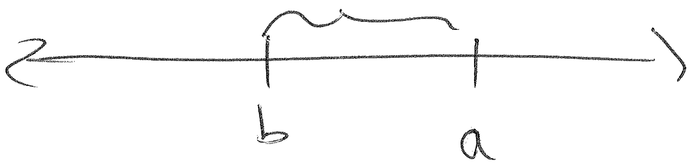
Geometrically



$|b-a| = |a-b| = d(a,b) = d(b,a)$.



$d(a,b) = d(b,a)$



A.3 Integer Exponents

Def$^{\underline{n}}$: If a is any real number and n is any integer, then the expression a^n means multiply a by itself n times. If $n=0$, then $a^0 = 1$.

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

Eg.: $3^2 = 3 \cdot 3 = 9$

$4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Arithmetic with Exponents

Let a be a real number and let m, n be integers. Then

$$a^m \cdot a^n = (\underbrace{a \cdot a \cdots a}_{m})(\underbrace{a \cdot a \cdots a}_{n})$$

$$= \underbrace{\underbrace{a \cdot a \cdots a}_{m} \cdot \underbrace{a \cdot a \cdots a}_{n}}_{m+n}$$

$$= a^{m+n}$$

Eg.: $2^3 \cdot 2^2 = (\underbrace{2 \cdot 2 \cdot 2}_{3})\ (\underbrace{2 \cdot 2}_{2})$

$= \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5}$

$= 2^5$

Negative Exponents

Let a be a real number, n a positive integer, then the expression

$$a^{-n} = \frac{1}{a^n}.$$

Eg.: $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$. $\quad 4^{-5} = \frac{1}{4^5}$ etc.

Rules for Working with Exponents

1. $a^m a^n = a^{m+n}$

2. $\dfrac{a^m}{a^n} = a^{m-n}$ $\qquad \left[\dfrac{a^m}{a^n} = a^m \cdot a^{-n} = a^{m-n}\right]$

3. $(a^m)^n = a^{mn}$ $\qquad \left[(a^m)^n = \underbrace{a^m \cdot a^m \cdots a^m}_{n} = a^{mn}\right]$

4. $(ab)^n = a^n b^n$ $\qquad \left[(ab)^n = \underbrace{(ab)(ab)\cdots(ab)}_{n} = \underbrace{a \cdots a}_{n}\,\underbrace{b \cdots b}_{n} = a^n b^n\right]$

5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
   $b \neq 0$

$$\left[\left(\frac{a}{b}\right)^n = \left(a\left(\frac{1}{b}\right)\right)^n = a^n\left(\frac{1}{b}\right)^n = a^n (b^{-1})^n = a^n b^{-n} = a^n\left(\frac{1}{b^n}\right) = \frac{a^n}{b^n}.\right]$$

E.g.: $\dfrac{a^n b^n}{b^n} = a^n b^n b^{-n}$

$= a^n b^{n+(-n)}$

$= a^n b^0$

$= a^n \cdot 1$

$= a^n$.

Eg: ① $7^5/7^3 = 7^{(5-3)} = 7^2 = 49$

② $\dfrac{3^6 \cdot 3^{-2}}{3} = \dfrac{3^6}{3^2 \cdot 3} = \dfrac{3^6}{3^3} = 3^{6-3} = 3^3$
$= 27.$

$\dfrac{3^6 \cdot 3^{-2}}{3} = \dfrac{3^{6+(-2)}}{3} = \dfrac{3^4}{3} = 3^{4-1} = 3^3$
$= \underline{27}$

③ $x^5 x^2 = x^{5+2} = x^7$

④ $\left(\dfrac{a^2}{2b}\right)^4 = \dfrac{(a^2)^4}{(2b)^4} = \dfrac{a^{2\cdot 4}}{2^4 b^4} = \underline{\dfrac{a^8}{16 b^4}}$.

A.4 Radicals & Rational Exponents

Def^n: If n is a positive integer and a is a real number, then

$$a^{1/n} = \sqrt[n]{a}.$$

When n is even, we require that $a \geq 0$.

E.g.: $\sqrt{4} = 2$ (because $2^2 = 4$)

When n is even, we require that $\sqrt[n]{a}$ is positive. This is called the principle n^{th} root.

<u>Rmk</u>: If n is an even number, then $(-1)^n = 1$, so ~~then~~ if $\sqrt[n]{a}$ is the positive/principle n^{th} root of a (i.e. $(\sqrt[n]{a})^n = a$), then

$$\begin{aligned}(-\sqrt[n]{a})^n &= (-1)^n(\sqrt[n]{a})^n \\ &= 1 \cdot a \\ &= a.\end{aligned}$$

<u>Rational Exponents</u>

<u>Recall</u>: A rational number is a ratio of two integers a/b such that a and b have no common factors.

E.g.: $1/2$, $2/3$, $5/9$, etc.

If a is any real number, then for two integers m/n, $n \neq 0$,

$$a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$$

when the n^{th} root is defined.

The same rules as above apply replacing "integer" with "rational number".

i.e. ~~Eg.~~ $a^{m/n} a^{s/t} = a^{m/n + s/t}$.

Eg: $16^{3/4} = (2^4)^{3/4}$

$$= ((2^4)^{1/4})^3$$

$$= 2^3$$

$$= 8.$$