MATH 111
EXAM 03

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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.**
You may use a calculator, but you may **not** use a Computer Algebra System (CAS) or any other electronic device whatsoever, **including cell phones.**

Name: Solutions

Problem	Points Earned	Points Possible
1		3
2		4
3		3
4		5
5		5
6		20
7		20
8		20
9		20
Bonus		10
Total		100

Date: April 22, 2015.

1. Definitions

1 (4 Points). *Let a be a fixed positive number. The base a logarithm of x is defined by*

$$\log_a(x) = y \text{ if and only if } \underline{x = a^y}.$$

2. *Let a be a positive number. Fill in the blanks.*

(a) $\log_a(1) = \underline{0}$.

(b) $\log_a(a) = \underline{1}$.

(c) $\log_a(a^x) = \underline{x}$.

(d) $a^{\log_a(x)} = \underline{x}$.

3. *Let $0 < a$ and C be fixed numbers. Fill in the blanks.*

(a) $\log_a(xy) = \underline{\log_a(x) + \log_a(y)}$.

(b) $\log_a\left(\frac{x}{y}\right) = \underline{\log_a(x) - \log_a(y)}$.

(c) $\log_a(x^C) = \underline{C\log_a(x)}$.

4. *Let a and b be fixed positive numbers. Use the Change of Base formula to rewrite $\log_a(x)$ with base b.*

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

5. *State the Horizontal Line Test.*

A function $f(x)$ has an inverse if and only if any horizontal line intersects the graph of $f(x)$ in at most one point.

2. Problems

6. *Let* $f(x) = 3x^2 - 6x - 9$.

(a) Find the solutions to $f(x) = 0$.

$0 = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3)$
$= 3(x+1)(x-3)$

so $x = -1$, $x = 3$.

(b) Write $f(x)$ *in vertex form.*

$h = \frac{-b}{2a} = \frac{-(-6)}{2(3)} = \frac{6}{6} = 1.$

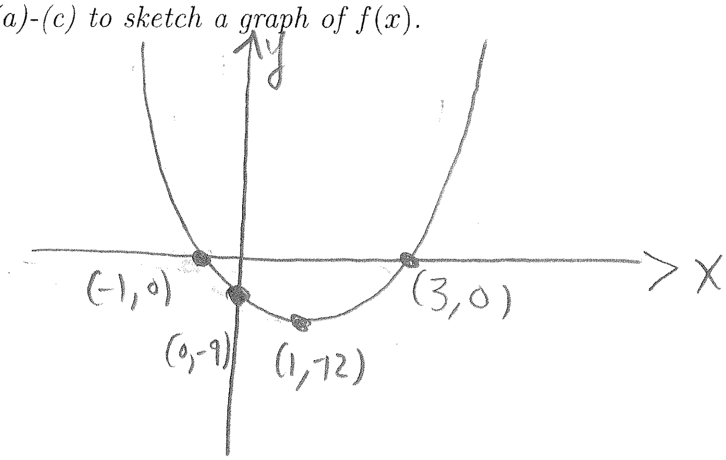
$k = f(h) = 3(1+1)(1-3) = 3(2)(-2) = 3(-4) = -12.$

$f(x) = a(x-h)^2 + k$
$= 3(x-(1))^2 + (-12)$
$= 3(x-1)^2 - 12.$

(c) Find the coordinates of the y-intercept for $f(x)$.

$f(0) = -9$, so the y-intercept is $(0, -9)$.

(d) Use parts (a)-(c) to sketch a graph of $f(x)$.



7. *Compute the following logarithms.*

(a) $\log_3(27)$.

$$\log_3(27) = \log_3(3 \cdot 9) = \log_3(3 \cdot 3^2) = \log_3(3^3) = 3.$$

(b) $\log_3(81)$.

$$\log_3(81) = \log_3(9^2) = \log_3((3^2)^2) = \log_3(3^4) = 4$$

(c) $\log_{16}(8)$.

$$\log_{16}(8) = \frac{\log_2(8)}{\log_2(16)} = \frac{\log_2(2^3)}{\log_2(2^4)} = \frac{3}{4}.$$

(d) $\log_{27}(81)$.

$$\log_{27}(81) = \frac{\log_3(81)}{\log_3(27)} = \frac{4}{3}.$$

8. *(a) Simplify the expression*

$$2\log_2(\sqrt{x+2}) - \log_2\left(\frac{1}{x-2}\right).$$

$$2\log_2(\sqrt{x+2}) - \log_2\left(\tfrac{1}{x-2}\right) = \log_2\left(\sqrt{x+2}^{\,2}\right) - \log_2\left((x-2)^{-1}\right)$$
$$= \log_2(x+2) - (-1)\log_2(x-2)$$
$$= \log_2(x+2) + \log_2(x-2)$$
$$= \log_2((x+2)(x-2)) = \log_2(x^2-4).$$

(b) Solve the following equation for x

$$2\log_2(\sqrt{x+2}) - \log_2\left(\frac{1}{x-2}\right) = 5$$

By (a) we have

$$\log_2(x^2-4) = 5$$
$$\Rightarrow 2^{\log_2(x^2-4)} = 2^5 = 32$$
$$\Rightarrow x^2-4 = 32 \Rightarrow x^2 = 36$$
$$\Rightarrow x = \pm 6.$$

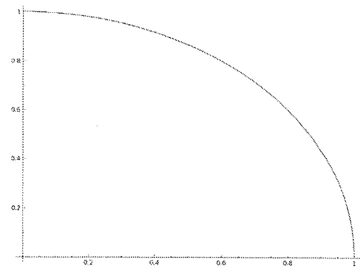
Note: $x=-6$ is an extraneous solution because both $-6+2=-4$ and $\frac{1}{-6-2} = \frac{-1}{8}$ are negative and $\sqrt{\ }$, $\log_2(\_)$ are only defined for positive numbers. So $x=6$ is the only solution.

9. *Solve the following equation for x*

$$2^{-4x} = 16 \cdot 2^{x^2}$$

$$2^{-4x} = 16 \cdot 2^{x^2}$$
$$\Rightarrow \log_2(2^{-4x}) = \log_2(16 \cdot 2^{x^2})$$
$$\Rightarrow -4x = \log_2(16) + \log_2(2^{x^2}) = 4 + x^2$$
$$\Rightarrow x^2 + 4x + 4 = 0$$
$$\Rightarrow (x+2)^2 = 0$$
$$\Rightarrow x = -2.$$

10 (Bonus). *Consider the function $f(x) = \sqrt{1-x^2}$ on the interval $[0,1]$. The graph of this function is given below. Is this function ivertible? If so, what is its inverse? Justify your answers.*



Yes, this function is invertible because it passes the horizontal line test.

To find the inverse, set $y = \sqrt{1-x^2}$ and solve for x.

$$y = \sqrt{1-x^2} \Rightarrow y^2 = 1 - x^2$$
$$\Rightarrow x^2 + y^2 = 1$$
$$\Rightarrow x^2 = 1 - y^2$$
$$\Rightarrow x = \sqrt{1-y^2}.$$

We disregard the solution $x = -\sqrt{1-y^2}$ because we have assumed that $0 \leq x \leq 1$.

So, this function is its own inverse!

Namely

$$f \circ f(x) = f\left(\sqrt{1-x^2}\right) = \sqrt{1-\left(\sqrt{1-x^2}\right)^2} = \sqrt{1-(1-x^2)} = \sqrt{1-1+x^2} = \sqrt{x^2} = x$$

whenever $0 \leq x \leq 1$.