MATH 111
EXAM 02

BLAKE FARMAN
UNIVERSITY OF SOUTH CAROLINA

Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.**
You may use a calculator, but you may **not** use a Computer Algebra System (CAS) or any other electronic device whatsoever, **including cell phones.**

Name: Answer Key

Problem	Points Earned	Points Possible
1		4
2		6
3		5
4		3
5		2
6		16
7		16
8		16
9		16
10		16
Bonus		5
Survey Bonus		5
Total		100

Date: March 25, 2015.

1. Definitions

1 (4 Points). *(a) State the Point-Slope form of a line passing through the point (x_1, y_1) with slope m.*

$$y - y_1 = m(x - x_1)$$

(b) State the Slope-Intercept form of a line with slope m and y-intercept b.

$$y = mx + b$$

2 (6 Points). *Let $f(x)$ be a function. State the average rate of change of f between $x = a$ and $x = b$.*

$$\frac{f(b) - f(a)}{b - a} \quad \text{or} \quad \frac{f(a) - f(b)}{a - b}$$

3 (5 Points). *If $f(x)$ is an exponential function with growth/decay factor a, express the growth/decay rate, r, in terms of the growth/decay factor.*

$a = 1 + r$

$\Rightarrow r = a - 1.$

4 (3 Points). *(a) State the general form of an exponential function.*

$f(x) = Ca^x$

(b) When does such a function model exponential growth?

When $a > 1.$

(c) When does such a function model exponential decay?

When $0 < a < 1.$

5 (2 Points). *Consider the two lines $f(x) = m_1x + b_2$ an $g(x) = m_2x + b_2$.*

(a) When are f and g parallel?

When $m_1 = m_2$

(b) When are f and g perpendicular?

When $m_1 m_2 = -1$ or, equivalently, when either $m_1 = \frac{-1}{m_2}$ or $m_2 = \frac{-1}{m_1}$.

2. Problems

6 (16 Points). *In the following problems, use the given information to find the equation of the line in slope-intercept form.*

(a) The line passing through the points $(3, 17)$ *and* $(6, 2)$.

$$m = \frac{17-2}{3-6} = \frac{15}{-3} = -5.$$

$$y - 2 = -5(x-6)$$
$$\Rightarrow y = -5x + 30 + 2$$
$$\Rightarrow y = -5x + 32.$$

(b) The line passing through the point $(2, 4)$ *and parallel to the line* $3y - 12x = 15$.

$$3y - 12x = 15$$
$$\Rightarrow 3y = 12x + 15$$
$$\Rightarrow y = 4x + 5$$
$$\Rightarrow m = 4$$

The line parallel to $3y-12x=15$ and passing through $(2,4)$ is

$$y - 4 = 4(x-2) \Rightarrow y = 4x - 8 + 4$$
$$\boxed{\Rightarrow y = 4x - 4}$$

(c) The line passing through the origin (that is, the point $(0, 0)$*) and perpendicular to the line* $3y - 12x = 15$. The slope of $3y - 12x = 15$ is 4, so a line perpendicular has slope $(\frac{-1}{4})$. Thus the equation of the desired line is

$$y = \frac{-1}{4}x + 0 = \frac{-1}{4}x.$$

7 (16 Points). *Consider the two lines* $f(x) = 2x + 9$ *and* $g(x) = -x + 3$. *Find the point (that is, the* (x, y) *pair) where these two lines intersect.*

$$2x + 9 = -x + 3$$
$$\Rightarrow 2x + x = 3 - 9$$
$$\Rightarrow 3x = -6$$
$$\Rightarrow x = -2$$

$$g(-2) = -(-2) + 3$$
$$= 2 + 3$$
$$= 5$$

So f and g intersect at $(-2, 5)$.

8 (16 Points). *Let* $f(x) = x^2 - 1$.

(a) Compute the average rate of change for f *between* $x = 3$ *and* $x = 5$.

$$\frac{f(5)-f(3)}{5-3} = \frac{(5^2-1)-(3^2-1)}{2}$$
$$= \frac{25-1-9+1}{2}$$
$$= \frac{25-9}{2} = \frac{16}{2} = 8.$$

(b) Give the Point-Slope form of the line that passes through $(3, f(3))$ *and* $(5, f(5))$.

$y - 8 = 8(x-3)$ or $y - 25 = 8(x-5)$

(c) Give the Slope-Intercept form of the line that passes through $(3, f(3))$ *and* $(5, f(5))$.

$y - 8 = 8(x-3)$ or $y - 25 = 8(x-5)$

$\Rightarrow y = 8x - 24 + 8$
$= 8x - 16$

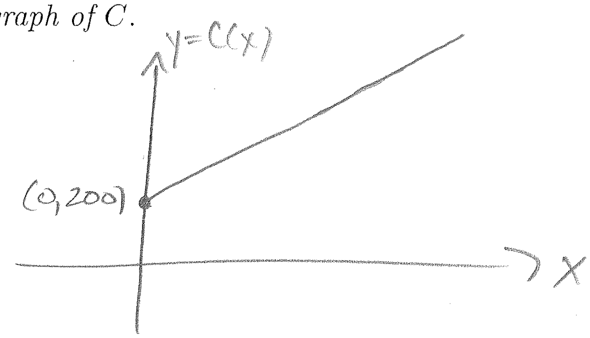
$\Rightarrow y = 8x - 40 + 25$
$= 8x - 16.$

9 (16 Points). *Bob is hosting an event. He is renting a facility, which costs* \$200, *and providing refreshments, which cost* \$6 *per guest.*

(a) Find a function, C, that models the total cost of the event if x people attend.

$C(x) = 6x + 200.$

(b) Sketch a graph of C.



(c) Evaluate $C(10)$ and $C(15)$. What do these numbers represent?

$C(10) = 6(10) + 200$
$= 60 + 200$
$= 260$

$C(15) = 6(15) + 200$
$= 90 + 200$
$= 290.$

The cost if 10 or 15 people attend, respectively.

(d) If the total cost for the event was \$500, *how many people attended?*

$500 = 6x + 200$

$\Rightarrow 300 = 6x$

$\Rightarrow x = \frac{300}{6} = 50.$

10 (16 Points). *A population of size* 16 *grows by* 25% *every day.*

(a) Give the daily growth factor for this population.

$r = 25/100 = \frac{1}{4}$ so $a = 1 + r = 1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$.

Equivalently, $a = 1 + .25 = 1.25$.

(b) Give an exponential model for the size of the population after t days.

$P(t) = 16\left(\frac{5}{4}\right)^t$

(c) Determine the size of the population after 2 days. [Hint: Express the growth factor as a fraction, rather than a decimal, and this will be very easy to compute.]

$$\begin{aligned} P(2) &= 16\left(\frac{5}{4}\right)^2 \\ &= 16\left(\frac{25}{16}\right) \\ &= 25. \end{aligned}$$

11 (Bonus - 5 Points). *Let $f(x) = x^2 - 1$. Show that the average rate of change of f between $x = a$ and $x = b$ is always $a + b$.*

$$\frac{f(b) - f(a)}{b - a} = \frac{(b^2 - 1) - (a^2 - 1)}{b - a}$$

$$= \frac{b^2 - 1 - a^2 + 1}{b - a}$$

$$= \frac{b^2 - a^2}{b - a}$$

$$= \frac{\cancel{(b - a)}(b + a)}{\cancel{(b - a)}}$$

$$= b + a.$$