MATH 111
EXAM 01

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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.**
You may use a calculator, but you may **not** use a Computer Algebra System (CAS) or any other electronic device whatsoever, **including cell phones.**

Name: Answer Key (Version 2)

Problem	Points Earned	Points Possible
1		3
2		6
3		2
4		3
5		3
6		3
7		16
8		16
9		16
10		16
11		16
Bonus		10
Total		100

Date: February 11, 2015.

1. Definitions

1 (3 Points). *Fill in the blanks with the correct factorizations.*

(a) $A^2 - B^2 =$ $(A+B)(A-B)$.

(b) $A^2 + 2AB + B^2 =$ $(A+B)^2$.

(c) $A^2 - 2AB + B^2 =$ $(A-B)^2$.

2 (6 Points). *Let a, b be non-zero real numbers and m, n integers. Fill in the blanks*

(i) $a^0 =$ 1,

(ii) $a^{-n} =$ $1/a^n$.

(iii) $a^m \cdot a^n =$ a^{m+n}

(iv) $\dfrac{a^m}{a^n}$ a^{m-n}

(v) $(a \cdot b)^n$ $a^n b^n$

(vi) $\left(\dfrac{a}{b}\right)^n$ a^n/b^n

3 (2 Points). *Given an equation $ax^2 + bx + c = 0$, the solutions are given by the Quadratic Formula. State the Quadratic Formula.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4 (3 Points). *Fill in the blanks:*

To make $x^2 + bx$ a perfect square, add $\left(\frac{b}{2}\right)^2$. *This gives the perfect square*

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = (x + b/2)^2.$$

5 (3 Points). *Fill in the blanks:*

An equation in variables x and y defines the variable y as function of the variable x if each value of x *corresponds to exactly* one *value of* y.

6 (3 Points). *If $f(x)$ is a function of a variable x, state the formula for the net change between the inputs $x = a$ and $x = b$, with $a \leq b$.*

$f(b) - f(a)$.

2. Problems

7 (16 Points). *Consider the equation*

$$y^2 + 3x = 9.$$

(a) Does this equation define y as function of x? Briefly justify why or why not. If it does, give the value of y when $x = 5/3$. No. When $x = 5/3$, we have

$$9 = y^2 + 3(5/3) = y^2 + 5$$

$\Rightarrow 4 = y^2$

$\Rightarrow y = \pm 2$, so there is a choice for the value of y given $x = 5/3$.

(b) Does this equation define x as function of y? Briefly justify why or why not. If it does, give the value of x when $y = 3$. Yes. Each value of y uniquely determines a value of x:

$$y^2 + 3x = 9 \Rightarrow 3x = 9 - y^2$$
$$\Rightarrow x = \frac{9 - y^2}{3}.$$

When $y = 3$, $x = \frac{9 - 3^2}{3} = \frac{9-9}{3} = \frac{0}{3} = 0.$

8 (16 Points). *Let $f(x) = x^2 + 3$. Compute the net change between $x = 3$ and $x = 5$.*

$$\begin{aligned} f(5) - f(3) &= (5^2 + 3) - (3^2 + 3) \\ &= (25 + 3) - (9 + 3) \\ &= 28 - 12 \\ &= 16. \end{aligned}$$

9 (16 Points). *Add the following rational expressions and simplify the result,*

$$\frac{2}{1-x^2}+\frac{1}{x+1}.$$

Observe that $1-x^2=(1-x)(1+x)$, so

$$\frac{2}{1-x^2}+\frac{1}{x+1}=\frac{2}{(1-x)(1+x)}+\frac{1}{x+1}$$

$$=\frac{2}{(1-x)(1+x)}+\frac{(1-x)}{(1-x)}\left(\frac{1}{x+1}\right)$$

$$=\frac{2+(1-x)}{(1-x)(1+x)}$$

$$=\frac{3-x}{(1-x)(1+x)}.$$

10 (16 Points). *Solve the equation*

$$x^2 - 4x + 2 = 0$$

for x. By the quadratic formula

$$X = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16-8}}{2}$$

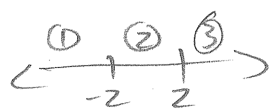
$$= \frac{4 \pm \sqrt{8}}{2}$$

$$= \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}.$$

11 (16 Points). *Solve the inequality*

$$0 \leq x^2 - 4$$

for x. Express the solution using interval notation and graph the solution on the number line. We have $x^2-4=(x-2)(x+2)$, so both 2 and -2 are solutions. We must test the three regions below:



Test points

$\underline{X=-3}$ $(-3)^2-4=9-4=5\geq 0$

$\underline{X=0}$

$0^2-4=-4<0,$

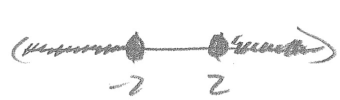
$\underline{X=3}$

$3^2-4=5\geq 0.$

These test points allow us to conclude that the inequality is satisfied when $x \leq -2$ or when $x \geq 2$. Therefore the solution is

$$(-\infty,-2] \cup [2,\infty)$$

or, graphically,



12 (Bonus - 10 Points). *Derive the Quadratic Formula, as stated in Exercise 3. [Hint: Use Exercise 4].*

Start from $ax^2+bx+c=0$, $a \neq 0$.

$$
\begin{aligned}
0 &= ax^2+bx+c \\
&= a\left(x^2+\tfrac{b}{a}x\right)+c \\
&= a\left(x^2+\tfrac{b}{a}x+\left(\tfrac{b}{2a}\right)^2-\left(\tfrac{b}{2a}\right)^2\right)+c \\
&= a\left(\left(x+\tfrac{b}{2a}\right)^2-\tfrac{b^2}{4a^2}\right)+c \\
&= a\left(x+b/2a\right)^2-\frac{b^2}{4a}+c
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow a\left(x+\tfrac{b}{2a}\right)^2 &= \frac{b^2}{4a}-c \\
&= \frac{b^2}{4a}-\frac{4ac}{4a} \\
&= \frac{b^2-4ac}{4a}
\end{aligned}
$$

$$\Rightarrow \left(x+b/2a\right)^2 = \frac{b^2-4ac}{4a^2}$$

$$
\begin{aligned}
\Rightarrow x+\frac{b}{2a} &= \pm\sqrt{\frac{b^2-4ac}{4a^2}} \\
&= \pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}} \\
&= \pm\frac{\sqrt{b^2-4ac}}{2a}
\end{aligned}
$$

$$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}. \quad \blacksquare$$