MATH 111
QUADRATICS WORKSHEET

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Name: ______________________________

Let $f(x) = -3x^2 + 6x + 9$. Use this function to answer questions Problems 1-3.

1. Write $f(x)$ in vertex form.

Solution. We can put $f(x)$ into vertex form by completing the square:

$$\begin{aligned} f(x) &= -3x^2 + 6x + 9 \\ &= -3(x^2 - 2x) + 9 \\ &= -3(x^2 - 2x + 1 - 1) + 9 \\ &= -3((x-1)^2 - 1) + 9 \\ &= -3(x-1)^2 + 3 + 9 \\ &= -3(x-1)^2 + 12. \end{aligned}$$

2. List in order the functions and transformations to get from the graph of $y = x^2$ to the graph of $y = f(x)$.

Proof. We start with the parabola, $y = x^2$.

- We obtain $y = (x-1)^2$ by shifting the parabola right by 1 unit,
- We obtain $y = 3(x-1)^2$ by stretching the graph of $y = (x-1)^2$ by a factor of three,
- We obtain $y = -3(x-1)^2$ by reflecting the graph of $y = 3(x-1)^2$ across the x-axis,
- Finally, we obtain $y = -3(x-1)^2 + 12 = f(x)$ by shifting the graph of $y = -3(x-1)^2$ up by 12 units.

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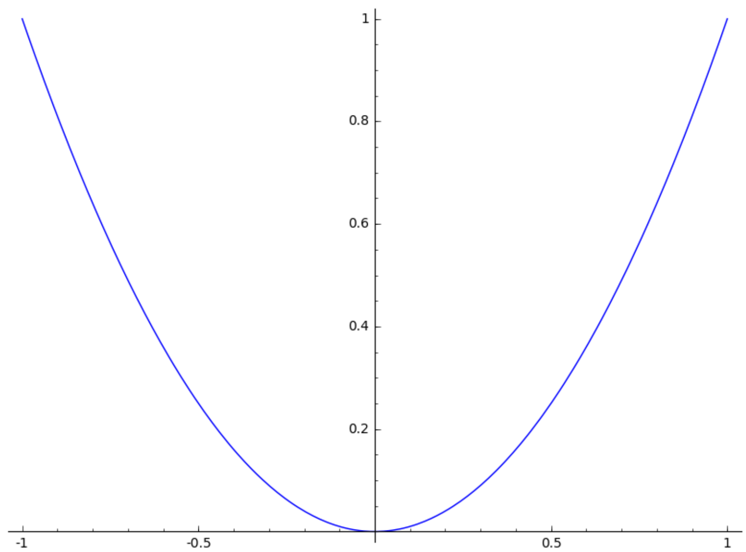
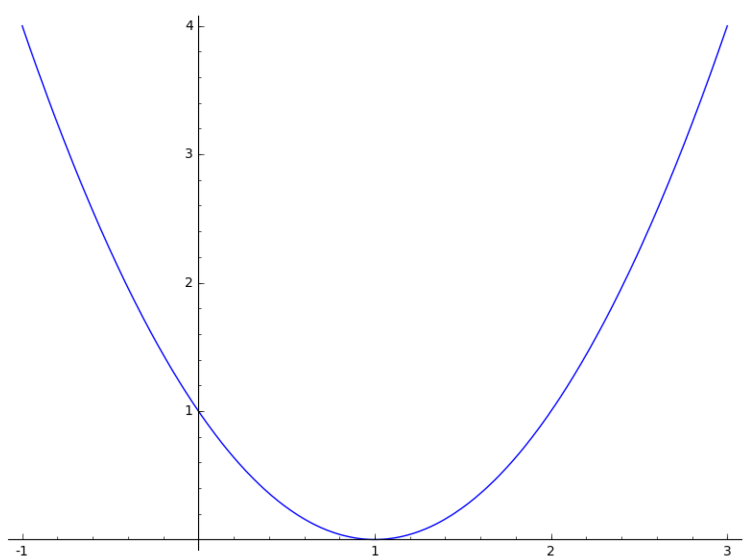
3. Starting from the graph of $y = x^2$ and ending at the graph of $y = f(x)$, sketch a graph of each of the functions, in order, from your answer to Problem 2. Label the vertex and y-intercept on each of your graphs.

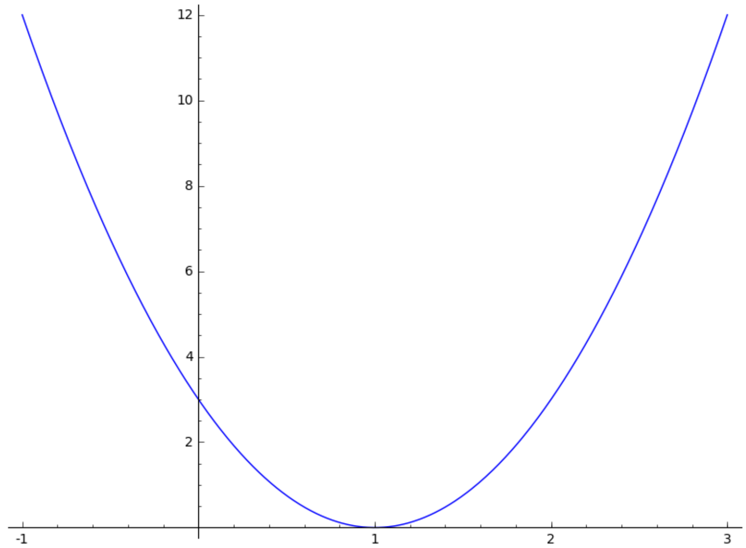
Solution. In the same order as above, the graphs are as follows:



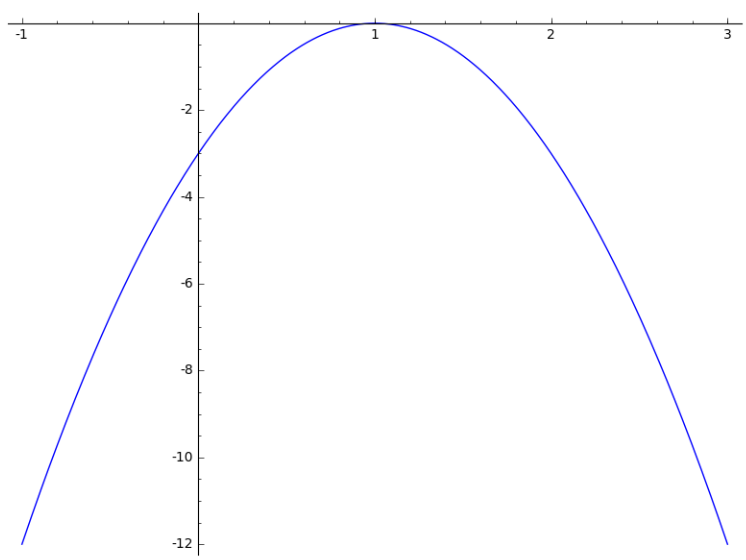
The vertex and y-intercept are both $(0, 0)$.



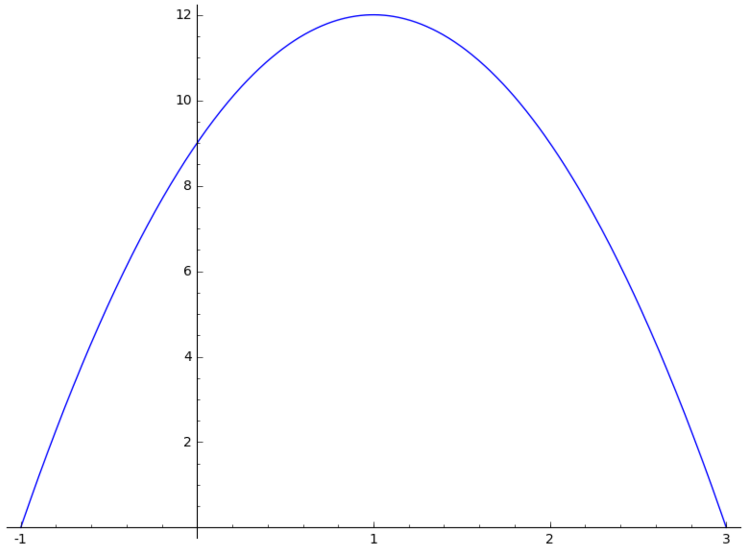
The vertex is $(1, 0)$ and the y-intercept is $(0, 1)$.



The vertex is $(1, 0)$ and the y-intercept is $(0, 3)$.



The vertex is $(1, 0)$ and the y-intercept is $(0, -3)$.



The vertex is $(1, 12)$ and the y-intercept is $(0, 9)$.

4. (a) Solve the equation

$$x^2 - 2x - 8 = 0$$

for x.

Solution. Factor

$$0 = x^2 - 2x - 8 = (x - 4)(x + 2)$$

to see that either $x = -2$ or $x = 4$.

(b) Find the (x, y)-coordinates of the y-intercept of

$$y = x^2 - 2x - 8.$$

Solution. The coordinates of the y-intercept are

$$(0, 0^2 - 2(0) - 8) = (0, 8).$$

(c) Find the (x, y)-coordinates of the vertex of

$$y = x^2 - 2x - 8.$$

Solution. The x-coordinate of the vertex is

$$h = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$$

and the y-coordinate of the vertex is

$$1^2 - 2(1) - 8 = 1 - 2 - 8 = -9.$$

Therefore the vertex is at

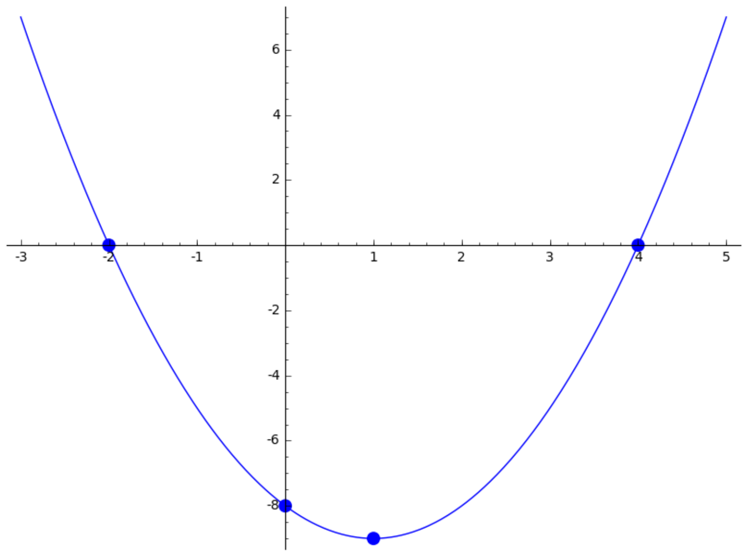
$$(1, -9).$$

5. Use your answers to Problem 4 to sketch a graph of

$$y = x^2 - 2x - 8.$$

Label the y-intercept, x-intercept(s), and the vertex.

Solution. The graph of $y = x^2 - 2x - 8$ is



The coordinates of the marked points, from left to right, are $(-2, 0)$, $(0, -8)$, $(1, -9)$, and $(4, 0)$.

Bob tosses a ball straight up in the air, then catches it. When he releases the ball, it leaves his hand at a velocity of 96 feet/sec. The height of the ball relative to Bob's hand after t seconds can be modeled by the function

$$h(t) = -16t^2 + 96t.$$

Use this model to answer the following Problems.

6. Sketch a graph of this function. Make sure that your graph is appropriate for this model.

Solution. To graph the function $h(t)$, we need the following points

- the y-intercept: $(0, h(0)) = (0, 0)$,
- the x-intercepts: Since $0 = h(t) = -16t^2 + 96t = -16t^2 + 16(6)(t) = -16t(t-6)$, they are

$$(0, 0) \text{ and } (3, 0),$$

- the vertex: The x-coordinate of the vertex is

$$\frac{-96}{2(-16)} = \frac{3(32)}{32} = 3$$

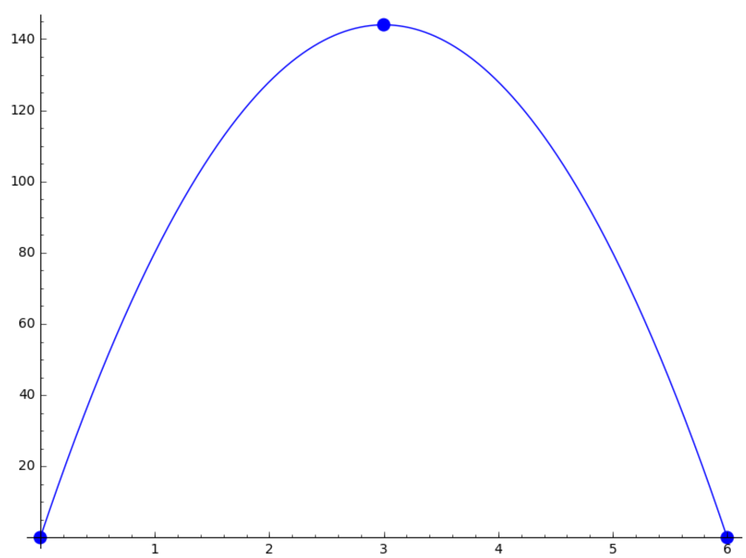
  so the y-coordinate is

$$h(3) = -16(9) + 96(3) = -16(9 - 18) = -16(-9) = 16(9) = 144.$$

  So the vertex is at

$$(3, 144).$$

The graph of $h(x)$ is then



Note that this model is only valid for $0 \leq t \leq 6$, since the height of zero at $t = 6$ implies that Bob has caught the ball, and its height is no longer governed by this model.

7. Use your answer from the previous problem to answer the following questions.

(a) After how many seconds does the ball reach its maximum height?

Solution. Looking at the graph, it's clear that the maximum occurs at the vertex, which is after 3 seconds have elapsed.

(b) What is the maximum height attained by the ball?

Solution. The height is the y-coordinate of the vertex, 144 feet.

(c) After how many seconds does the ball return to Bob's hand?

Solution. The ball returns to Bob's hand after 6 seconds; this is the other x-intercept.