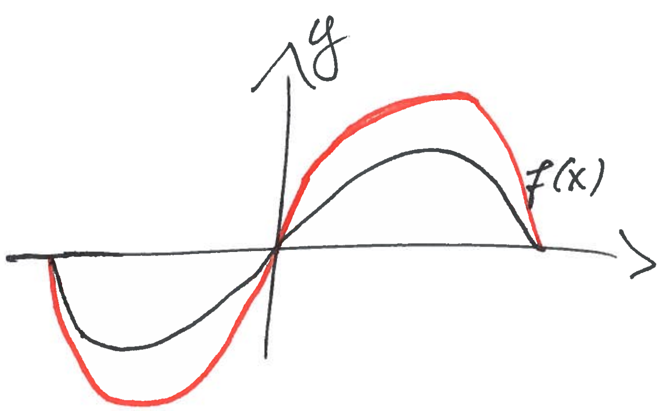
Vertical Stretching/Shrinking

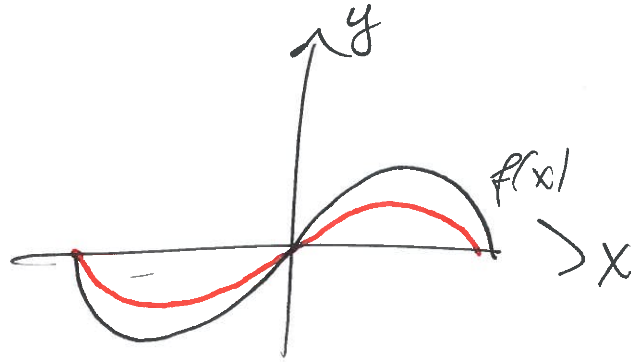
11/16/17 ①

$c > 0$, function $f(x)$, the graph of $cf(x)$ is

- If $1 < c$, the graph of $f(x)$ stretched by a factor of c (vertically)
- If $0 < c < 1$, the graph of $f(x)$ shrunk by a factor of c. (vertically)

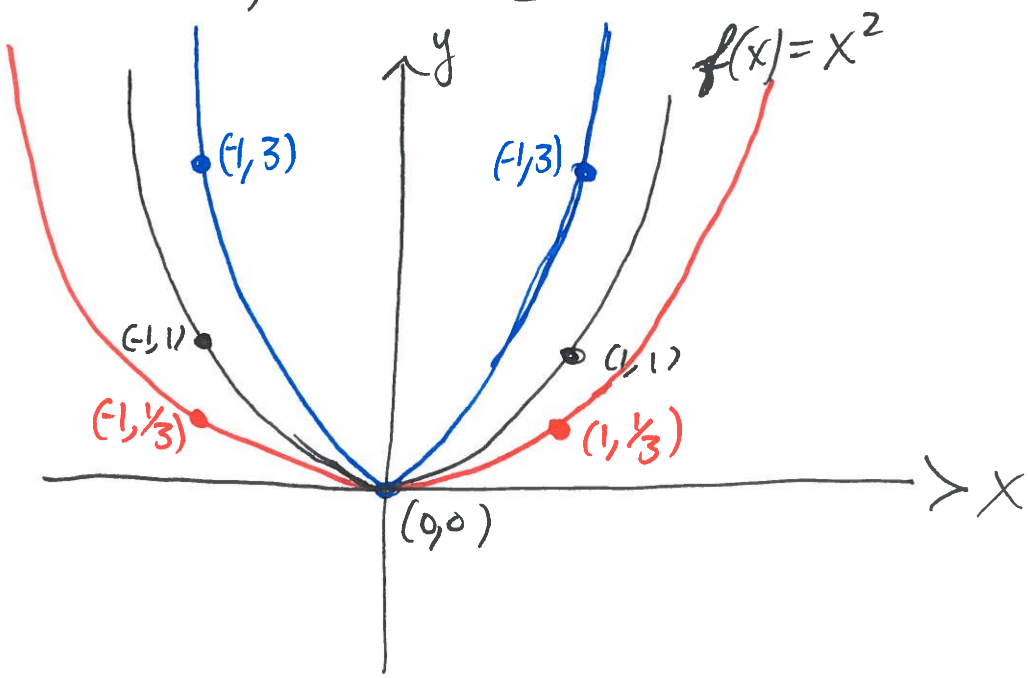


$1 < c$



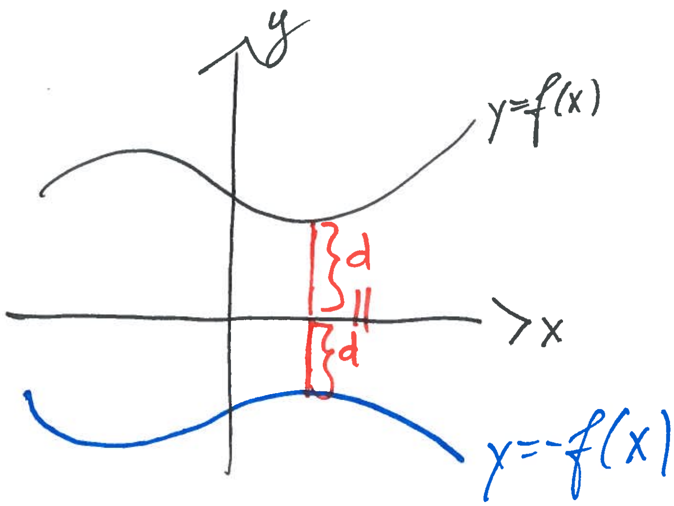
$0 < c < 1$

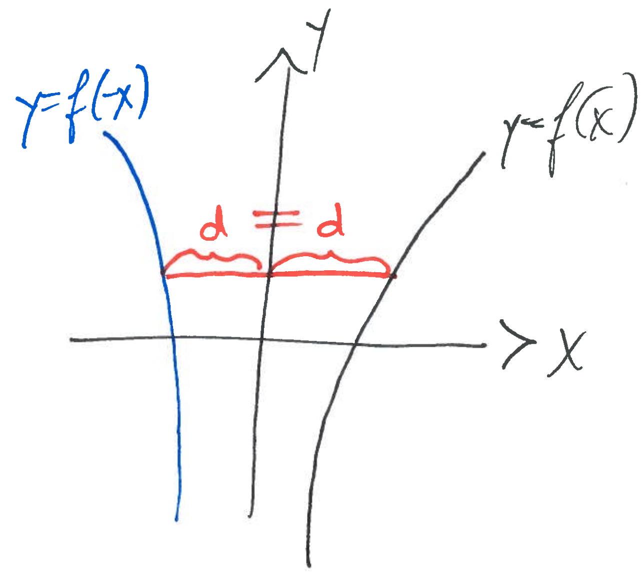
E.g.: $g(x) = 3x^2$, $h(x) = \frac{1}{3}x^2$



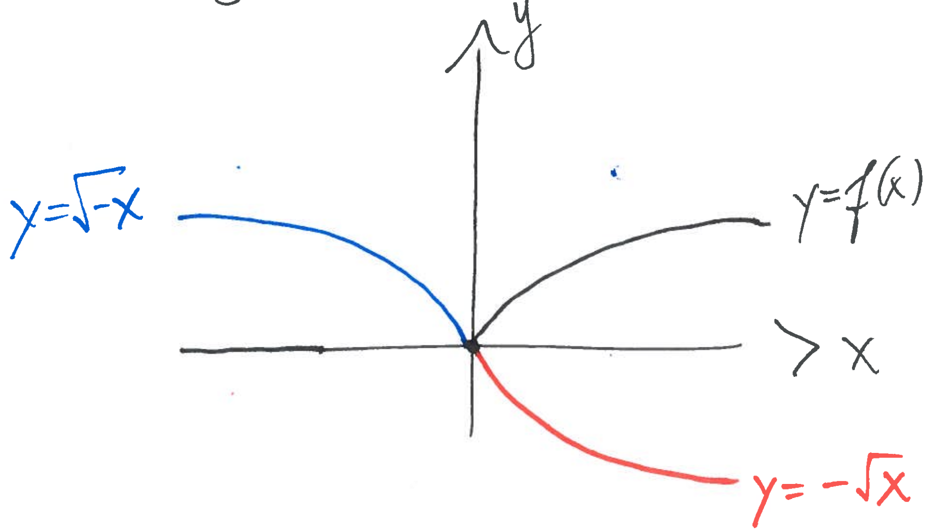
Reflection

- The graph of $y = -f(x)$ is the reflection of the graph of $f(x)$ across the x-axis,
- The graph of $y = f(-x)$ is the reflection of the graph of $f(x)$ across the y-axis.





E.g: $f(x) = \sqrt{x}$, $g(x) = \sqrt{-x}$



Graphing Quadratics

$$f(x) = ax^2 + bx + c$$

$(x+A)^2 = x^2 + 2Ax + A^2$

Know what the graph of $y = x^2$ looks like.

Complete the square:

$$f(x) = ax^2 + bx + c \text{, factor out } a \text{ from first two terms}$$

$$= a\underbrace{\left(x^2 + \tfrac{b}{a}x\right)}_{\text{complete the square on this binomial}} + c$$

$$= a\Big(\underbrace{x^2 + 2\left(\tfrac{b}{2a}\right)x + \left(\tfrac{b}{2a}\right)^2}_{\text{factor}} - \left(\tfrac{b}{2a}\right)^2\Big) + c$$

$$= a\left(\left(x + \tfrac{b}{2a}\right)^2 - \left(\tfrac{b}{2a}\right)^2\right) + c \text{, distribute the } a$$

$$= a\left(x + \tfrac{b}{2a}\right)^2 - a\left(\tfrac{b^2}{4a^2}\right) + c$$

$$= a\left(x + \tfrac{b}{2a}\right)^2 + \underbrace{\left[c - \tfrac{b^2}{4a}\right]}_{\text{rename } k}$$

$$= a\left(x + \tfrac{b}{2a}\right)^2 + k$$

Every degree 2 polynomial can be written in this form.

Know graph of $y = x^2$, so know the graph of $\left(x + \frac{b}{2a}\right)^2$ ← this is a horizontal translation of the parabola.

We obtain the graph of

$$a\left(x+\frac{b}{2a}\right)^2$$

by

1. stretching ~~the gr~~ or shrinking the graph of $\left(x+\frac{b}{2a}\right)^2$ by $|a|$,
2. if $a<0$, then reflect. Else, do nothing.

Finally, obtain the graph of

$$f(x)=ax^2+bx+c=a\left(x+\frac{b}{2a}\right)^2+k$$

by translating the graph of

$$a\left(x+\frac{b}{2a}\right)^2$$

by k.

- Horizontal shift,
- Stretch/shrink (vertical)
  - reflect if $a<0$
- Vertical Shift.

Corollary: The vertex of $f(x)=ax^2+bx+c$ has x-coordinate

$$-\frac{b}{2a}.$$

The value k above is just

$$f\left(-\frac{b}{2a}\right)=a\left(\frac{-b}{2a}+\frac{b}{2a}\right)^2+k=k.$$

Standard Form of a Quadratic

$$\left(f(x) = ax^2 + bx + c\right)$$

$$f(x) = a(x-h)^2 + k; \quad h = \frac{-b}{2a}, \quad k = f(h) = f\left(\frac{-b}{2a}\right).$$

The vertex of f is at (h,k).

Eg.: $f(x) = 2x^2 - 12x + 23$

Put in standard form:

$$f(x) = 2(x^2 - 6x) + 23$$

$6 = 2 \cdot 3$

$(x-3)^2 = x^2 - 2(3)x + 3^2$
$= x^2 - 6x + 9$

$$= 2(\underbrace{x^2 - 6x + 9} - 9) + 23$$

$$= 2\left((x-3)^2 - 9\right) + 23$$

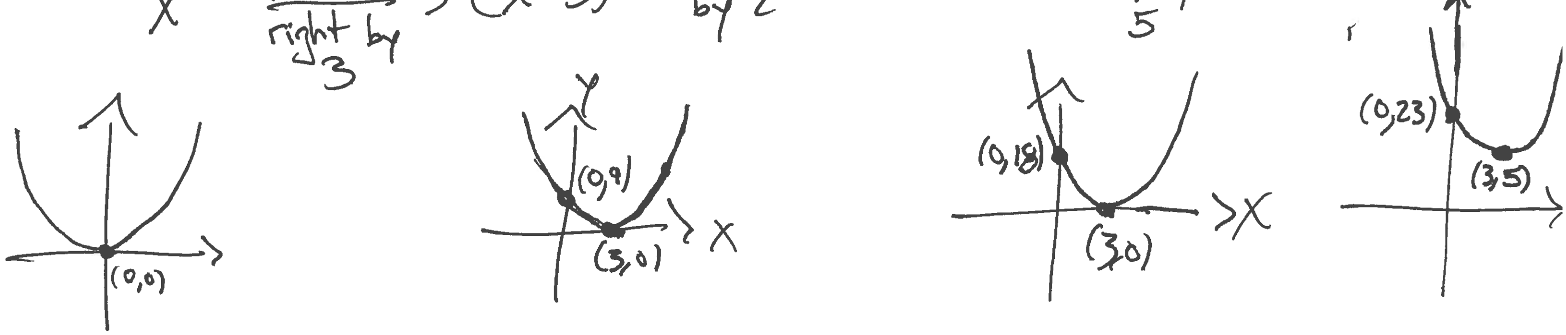
$$= 2(x-3)^2 - 18 + 23$$

$$= 2(x-3)^2 + 5$$

$h = 3, \ k = 5$

Graph:

$$x^2 \xrightarrow[\text{right by 3}]{\text{horiz. transl.}} (x-3)^2 \xrightarrow[\text{by 2}]{\text{vert. stretch}} 2(x-3)^2 \xrightarrow[\text{up by 5}]{\text{vert. transl.}} 2(x-3)^2 + 5$$



Quick & Dirty

There are at most 4 interesting points on a parabola. They are:

- The y-intercept (always there)
  i.e. $(0, f(0))$
- The vertex: $(h,k) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
- The root(s): the solutions to
  $$f(x) = ax^2+bx+c = 0.$$
  – Three options:

1 root	$b^2-4ac=0$
2 roots	$b^2-4ac>0$
0	$b^2-4ac<0$

Eg: $f(x) = 2x^2 - 12x + 23$

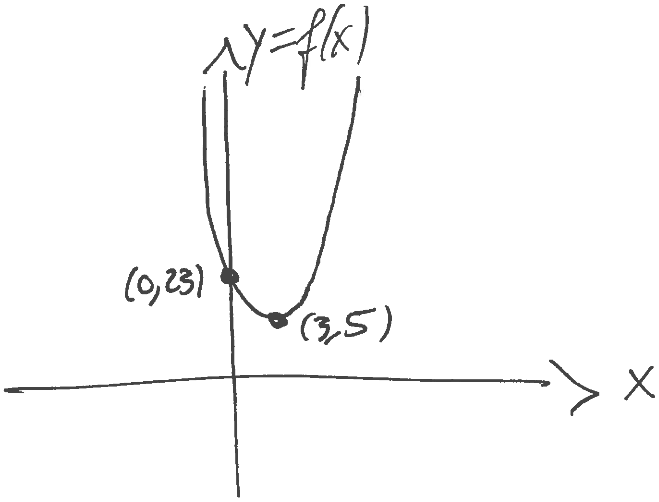
y-int: $(0, 23)$

vertex: $h = \frac{-(-12)}{2(2)} = \frac{12}{4} = 3$, $k = f(3) = 2(9) - 36 + 23$
$= 18 - 36 + 23$
$= 18 - 13$
$= 5.$

$(3, 5)$

<u>roots:</u> $(-12)^2 - 4(2)(23) = 144 - 2(92) = 144 - 184 < 0$

There are no real roots.



E.g.: $f(x) = x^2 + 16x + \underline{24}$

$2^4 \; 2^3$

y-int: (0,24)

$h = \frac{-16}{2(1)} = -8$; $k = f(-8) = (-8)^2 + 16(-8) + 24$

$= 64 - 128 + 24$

$= -64 + 24$

$= -40.$

vertex: (-8,-40)

$160+60+6$

226

$80+16=96$

$$x = \frac{-16 \pm \sqrt{16^2 - 4(24)}}{2}$$

$$= \frac{-16 \pm \sqrt{226-96}}{2} = \frac{-16 \pm \sqrt{130}}{2} \; ?$$