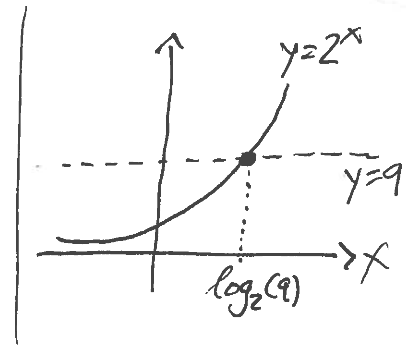
4.5: 11/14/17 ①.

E.g: Solve $2^x = 9$ for x.

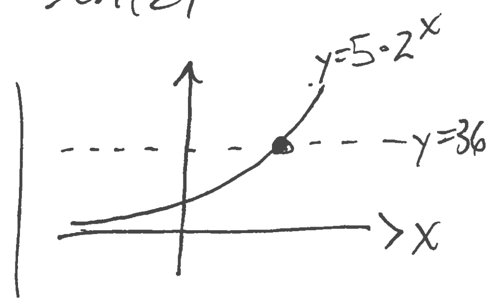


Apply $\log_2$ to both sides:

$$\log_2(2^x) = \log_2(9)$$

$$\Rightarrow x = \log_2(9) = \frac{\log(9)}{\log(2)} = \frac{\ln(9)}{\ln(2)}$$

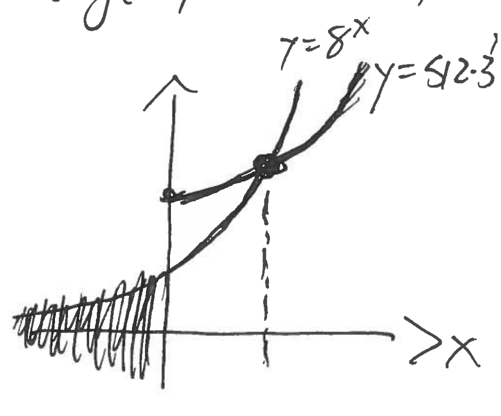
E.g.: Solve $5 \cdot 2^x = 36$ for x.



$$\Rightarrow 2^x = \frac{36}{5}$$

$$\Rightarrow x = \log_2(2^x) = \log_2\left(\frac{36}{5}\right) = \frac{\log\left(\frac{36}{5}\right)}{\log(2)} = \frac{\ln\left(\frac{36}{5}\right)}{\ln(2)}$$

E.g.: Solve $8^x = 512 \cdot 3^x$ for x



Divide both sides by 3^x:

$$512 = \frac{8^x}{3^x} = \left(\frac{8}{3}\right)^x$$

$$\Rightarrow \ln(512) = \ln\left(\left(\frac{8}{3}\right)^x\right) = x \ln\left(\frac{8}{3}\right)$$

$$\Rightarrow x = \frac{\ln(512)}{\ln(8/3)} \approx 6.36.$$

E.g.: Solve $e^{3-2x}=4$ for x.

$$\left[f(x)=3-2x,\ g(x)=e^x,\ e^{3-2x}=g\circ f(x) \right]$$

$$\ln\left(e^{3-2x}\right)=\ln(4)$$

$$\underset{3-2x}{\shortparallel}$$

$$\Rightarrow -2x=\ln(4)-3$$

$$\Rightarrow x=\frac{\ln(4)-3}{-2}=\boxed{\frac{3-\ln(4)}{2}}\approx 0.807.$$

E.g.: Solve $2^{x^2}=16$ for x.

$$\log_2\left(2^{x^2}\right)=x^2=\log_2(16)=4$$

Take the square root of both sides:

$$x=\pm 2.$$

E.g.: Solve $2\log(x)=3$ for x.

$$\log(x)=\frac{3}{2}$$

$$x=10^{\log(x)}=10^{3/2}\approx 31.62$$

E.g.: Solve $\ln(x+1)+\ln(5)=1$. for x.

$$\ln\left((x+1)5\right)=1$$

$$5(x+1) = e^{\ln(5(x+1))} = e$$

$$\Rightarrow x+1 = \frac{e}{5}$$

$$\Rightarrow x = \frac{e}{5} - 1 \approx -0.46$$

E.g.: $\log(x+1) - \log(x) = 2$: solve for x

$$\log\left(\frac{x+1}{x}\right) = 2$$

$$\frac{x+1}{x} = 10^{\log\left(\frac{x+1}{x}\right)} = 10^2$$

Multiply both sides by x:

$$x+1 = 10^2 x$$

Subtract x from both sides:

$$1 = 100x - x = 99x$$

Divide both sides by 99:

$$\boxed{x = \frac{1}{99}.}$$

$$\frac{12}{100}\left(\frac{1}{2}\right) = \frac{6}{100} = .06$$

E.g.: $P = 10{,}000$, $r = 12\%$, $n = 2$

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} = 10{,}000\left(1 + \frac{.12}{2}\right)^{2t}$$

$$= 10{,}000(1 + .06)^{2t}$$

$$= 10{,}000(1.06)^{2t}$$

How long will it take for the investment to grow to \$25,000?

Fancy way of asking: solve

$$10000(1.06)^{2t} = 25000$$

for t.

Divide both sides by 10000:

$$(1.06)^{2t} = \frac{25000}{10000} = 2.5$$

$$\Rightarrow \ln\left((1.06)^{2t}\right) = \ln(2.5)$$

$$\Rightarrow 2t\ln(1.06) = \ln(2.5)$$

$$\Rightarrow t = \frac{\ln(2.5)}{2\ln(1.06)} \approx 7.863;$$ so roughly 8 years.

5 Quadratic Functions and Models.

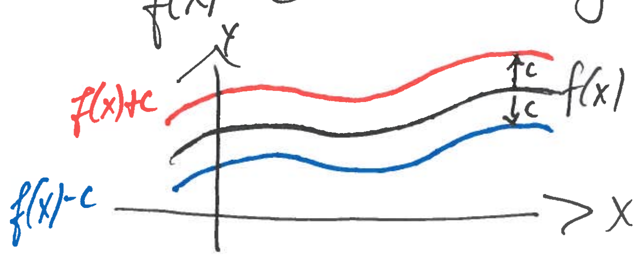
5.1: Shifting & Stretching

Vertical Shifting:

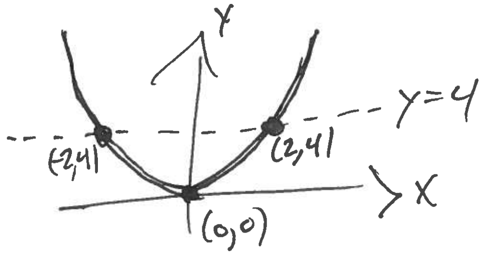
Suppose $c>0$, $f(x)$ is a function. The graph of

$f(x)+c$ is the graph of $f(x)$ shifted up c units

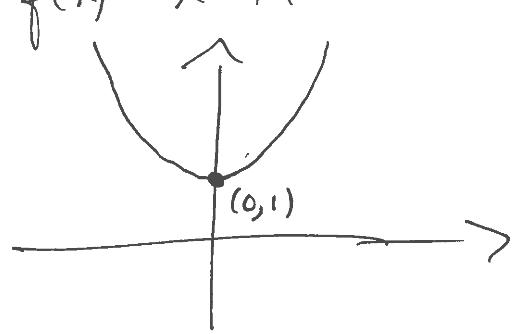
$f(x)-c$ is the graph of $f(x)$ shifted down c units.



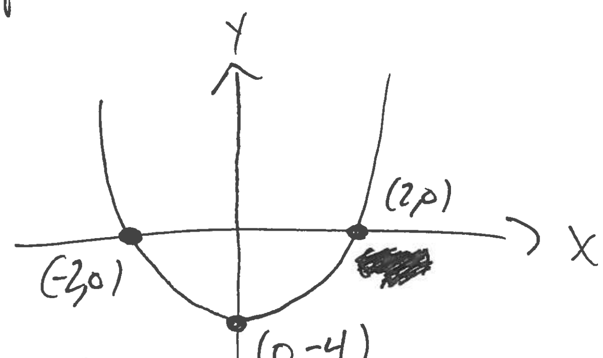
E.g: $f(x) = x^2$



$f(x) = x^2 + 1$



$f(x) = x^2 - 4$



One can think of translating vertically as moving the graph up by c ($f(x)+c$) or down by c ($f(x)-c$). Equivalently, one can think of moving the x-axis down by c ($f(x)+c$) or up by c ($f(x)-c$).

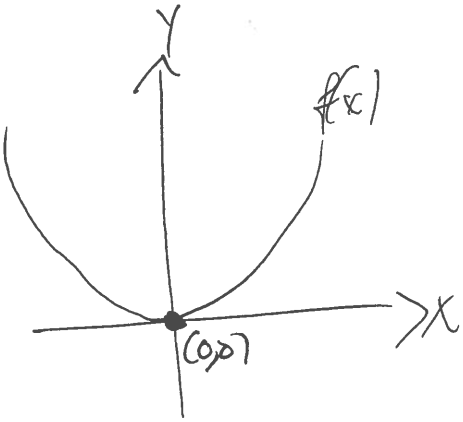
Horizontal Shifts:

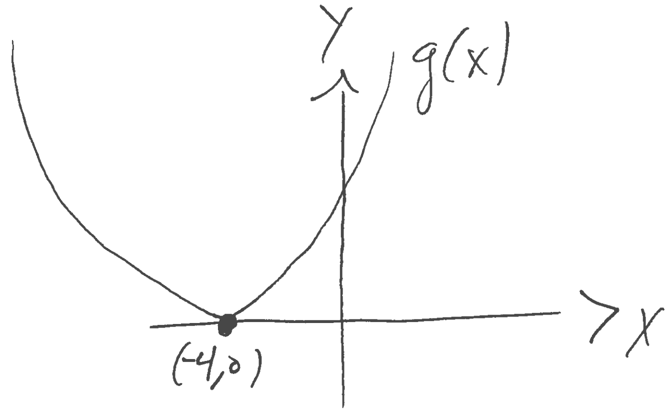
Suppose $c>0$, $f(x)$ is a function. The graph of $f(x+c)$ is the graph of $f(x)$ shifted left by c units.

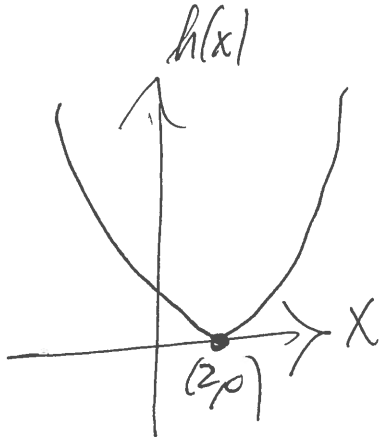
$f(x-c)$ is the graph of $f(x)$ shifted right by c units.

E.g.: $f(x) = x^2$

$g(x) = (x+4)^2, \; h(x) = (x-2)^2$







Think of

$f(x+c)$ – graph of f shifted left by c

or – shifted the y-axis right by c.

$f(x-c)$ – graph of f shifted right by c or

– shifted the y-axis left by c.

~~Ex~~ Consider the function

$$f(x) = x^2 + bx + c$$

$$\boxed{\begin{aligned} &ax^2+bx+c \\ &\quad \shortparallel \\ &a(x-h)^2+k \\ &\text{vertex form} \end{aligned}}$$

Complete the square:

$$f(x) = \underbrace{x^2 + 2\left(\tfrac{b}{2}\right)x + \left(\tfrac{b}{2}\right)^2} - \left(\tfrac{b}{2}\right)^2 + c$$

$$= \left(x + \tfrac{b}{2}\right)^2 + k \qquad k = c - \left(\tfrac{b}{2}\right)^2$$

E.g.: $f(x) = x^2 + 4x + 2$

$$f(x) = \left(x + \tfrac{4}{2}\right)^2 + \left(2 - \left(\tfrac{4}{2}\right)^2\right)$$
$$= (x+2)^2 + (2-4)$$
$$= (x+2)^2 - 2.$$

$$x^2, \xrightarrow{\text{shift left by 2}} (x+2)^2, \xrightarrow{\text{shift down by 2}} f(x) = \cancel{x^2}(x+2)^2 - 2$$