Increasing/Decreasing Functions

Let f be a function.

(i) The function, f, is strictly increasing on the interval, I, if for each $a, b \in I$ such that $a < b$, then

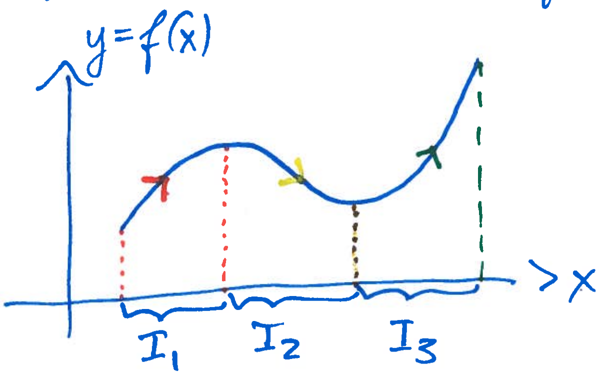
$$f(a) < f(b)$$

[If $f(a) \leq f(b)$, then one says f is non-decreasing on I]

(ii) The function, f, is strictly decreasing on the interval, I, if for each $a, b \in I$ such that $a < b$, then

$$f(b) < f(a)$$

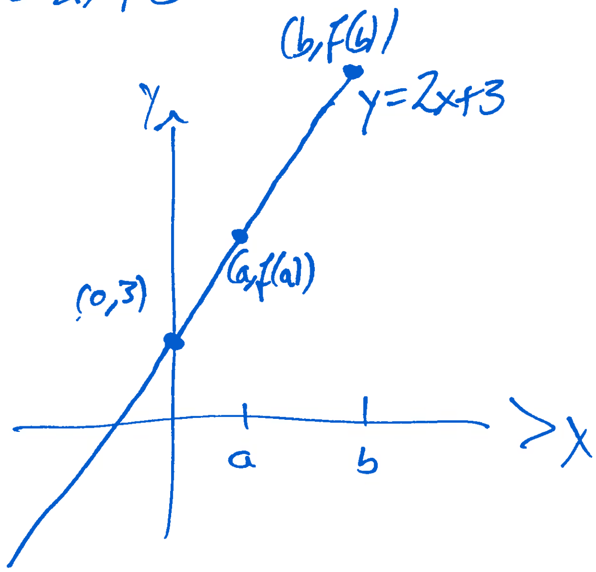
[If $f(b) \leq f(a)$, then one says f is non-increasing on I]



f is increasing on I_1,
f is decreasing on I_2,
f is increasing on I_3.

If f is an increasing/decreasing function on its domain, then we simply say f is increasing/decreasing.

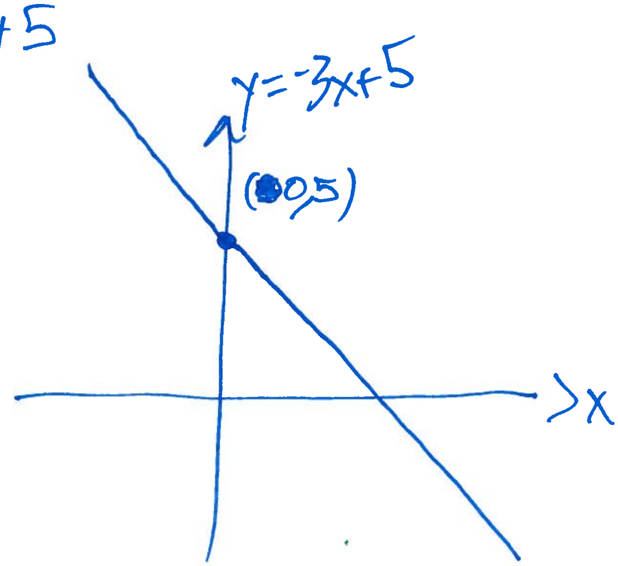
Eg: $f(x) = 2x+3$



This is an increasing function.

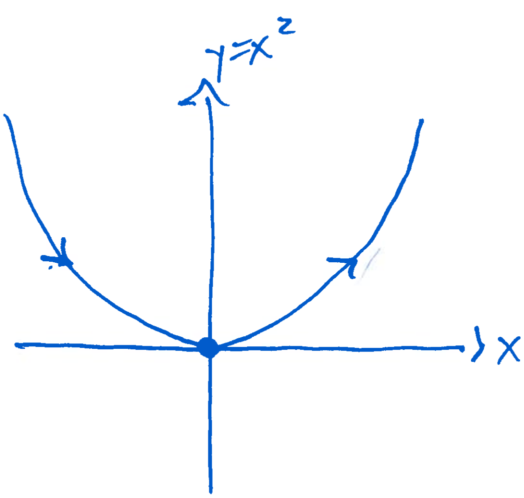
$(a<b \Rightarrow f(a)<f(b))$

Eg: $y = -3x+5$



This is an increasing function.

Eg: $y=x^2$



$y=x^2$

increasing on $[0,\infty)$

decreasing on $(-\infty,0]$.

Local Extrema

A function f has a <u>local</u> <u>maximum</u> at $x=a$ if

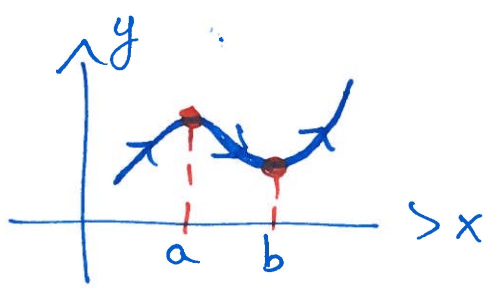
$$f(a) \geq f(x)$$

for all values of x near a.

A function f has a <u>local</u> <u>minimum</u> at $x=a$ if

$$f(a) \leq f(x)$$

for all values of x near a.



f has a local maximum at $x=a$, and a local minimum at $x=b$.

<u>Remark</u>: A local maximum always occurs at a point where the function switches from increasing to decreasing.

A local minimum always occurs at a point where the function switches from decreasing to increasing.

1.8 Working with Functions: Modeling Real World Relationships.

Recall: A model is a mathematical representation of a real world situation.

The process of creating a model is called modeling.

Eg.: A company manufactures baseball caps with school logos. The company charges their customers a fixed fee of \$500 for setting up the machines, and \$8 for each cap.

a) Find a linear model for purchasing any number of caps. Express this model in function form.

b) Use the model to find the cost of purchasing 225 caps.

Say that C stands for cost, and n stands for the number of caps produced. Here, C depends on n.

Identify the fixed cost of \$500 represents the value

$$C(0) = 500.$$

Every time you buy a cap, the cost goes up by \$8, this says the cost function is

$$C(n) = 8n + 500.$$

b) $C(225) = 8(225) + 500$

$= 2300.$

It will cost \$2,300 to obtain 225 caps.

The average U.S. resident uses 650 pounds of paper per year. The average pine tree produces 4130 pounds of paper.

(a) Find a function, N, that models the number of trees used for paper in one year used by x residents.

(b) The city of Cleveland Heights, OH, had a population of about 49,000 in 2003. Use the model to find the number of trees used to supply the residents with paper in 2003.

a) The number of trees used by each resident is the ratio of the # of lbs ~~thes~~ of paper used by the resident over the # of lbs of paper the tree produces.

We compute this value because

$$N = x \cdot (\#\text{of trees used by each resident})$$

$$= x \cdot \frac{650}{4130} \approx x(0.157)$$

b) $N(49000) = \underline{49000}\left(\frac{650}{4130}\right) \approx 7693.$

Eg: A gardener has a 1200-gallon water tank. During the spring, the gardener requires 80 gallons of water per day,

(a) Find a function w that gives the amount of water in the tank after x days.

$$w(x) = 1200 - 80x$$

b) How much water is in the tank after 3 days? 12 days?

$w(3) = 1200 - 80(3) = 1200 - 240 = 960$ gallons.

$$W(12) = 1200 - 80(12)$$
$$= 1200 - 800 - 160$$
$$= 1200 - 960$$
$$= 240 \text{ gallons.}$$

c) How many gallons are in the tank after 20 days?

$$W(20) = 1200 - 80(20)$$
$$= 1200 - 1600$$
$$= -400 \text{ gallons} \ldots \text{oops?}$$

Physically, this answer is nonsense: negative gallons don't have any physical meaning. This means that 20 is <u>not</u> in the domain of my model.

Want to restrict this model only to #of days such that

$$W(x) \geq 0$$

Solve

$$1200 - 80x = 0$$
$$\Rightarrow 1200 = 80x$$
$$\Rightarrow x = \frac{1200}{80} = 15$$

So the model is only valid for $0 \leq x \leq 15$. Fix the model

$$W(x) = \begin{cases} 1200 - 80x & 0 \leq x \leq 15, \\ 0 & 15 < x. \end{cases}$$