$f(x) = \sqrt{x^2+2x-3}$ find the domain. 10/3/17 (1)

Domain of f is all x such that

$$0 \leq x^2 + 2x - 3.$$

$$0 \leq (x+3)(x-1)$$

$(-\infty, -3] \cup [1, \infty)$

Test $x = -4$:

$$-4^2 + 2(-4) - 3 = (-4+3)(-4-1) = (-1)(-5) = 5 \geq 0$$

Test $x = 2$

$$(2)^2 + 2(2) - 3 = (2+3)(2-1) = (5)(1) = 5 \geq 0$$

$$0^2 + 2(0) - 3 = -3 < 0$$

Extra Credit

$p(x) = x^2 + 5x + 6$

a) Find a & b such that the net change in p from a to b is 6

b) Find $c < d$ such that " " " " " " c to d is -6.

---

a) Net change

$$f(b) - f(a) = 6$$

Find b such that $f(b) = 6$, find a such that $f(a) = 0$. Find solutions to

$$f(x) = x^2 + 5x + 6 = 6 \iff x^2 + 5x = 0$$

and $x^2 + 5x + 6 = 0$

$x(x+5) = 0$ $\qquad$ $(x+2)(x+3) = 0$

$x = 0, x = -5$ $\qquad$ $x = -2, x = -3$

$f(0) = 6, \quad f(-2) = f(-3) = 0$

Take $b = 0$, $a = -2$ (or $a = -3$)

b) Want $f(c) = 6$, $f(d) = 0$ $\quad (c < d)$

Net change: $f(d) - f(c) = -6$ $\qquad$ Take $c = -5$, $d = -2$ or -3.

1.6 Working with functions: graphs & graphing calculators.

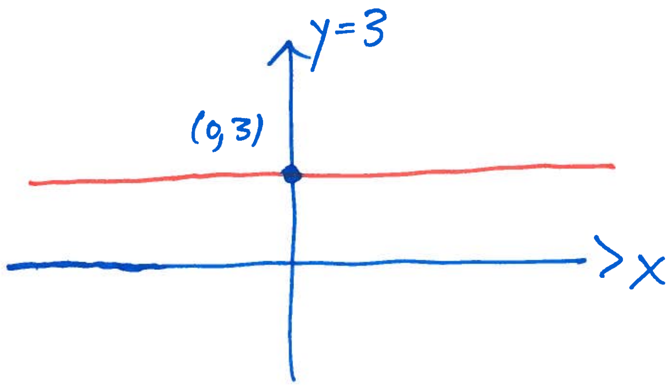
Constant functions: for some number a,

$$f(x) = a$$

E.g.: $f(x) = 3$.

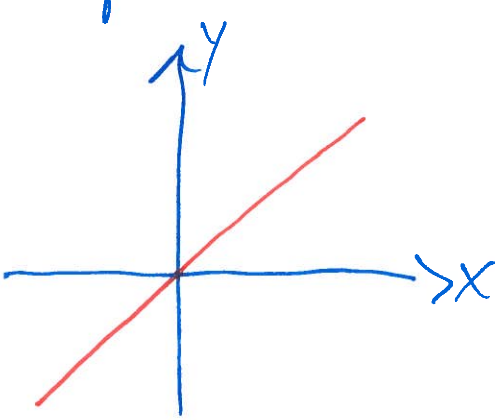
$f(1) = 3$, $f(2) = 3$, $f(\pi) = 3$, etc.

Graph of such a function is a horizontal line.

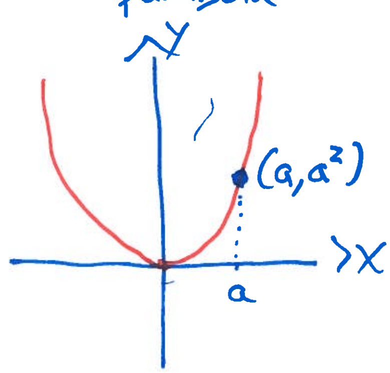


Basic Functions

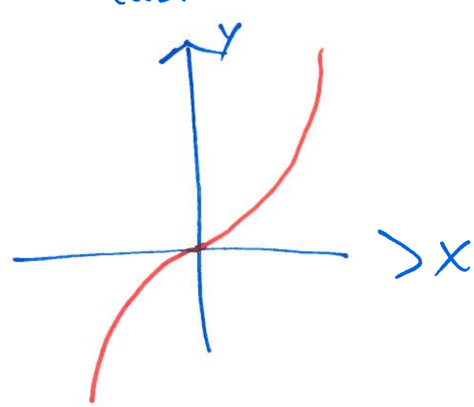
$f(x) = x$ line



$f(x) = x^2$ parabola

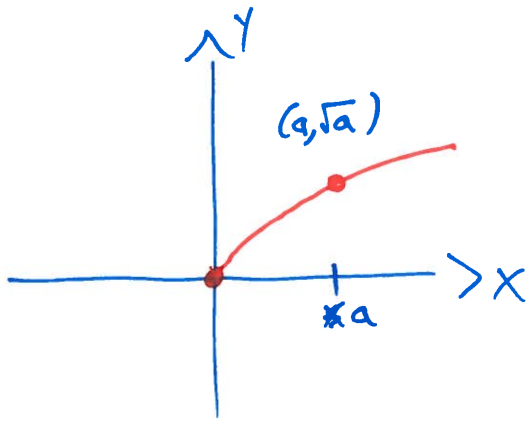


$f(x) = x^3$ cubic



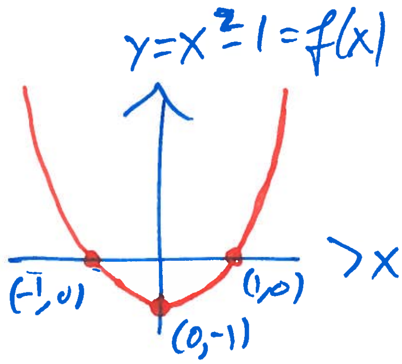
Square Root Function

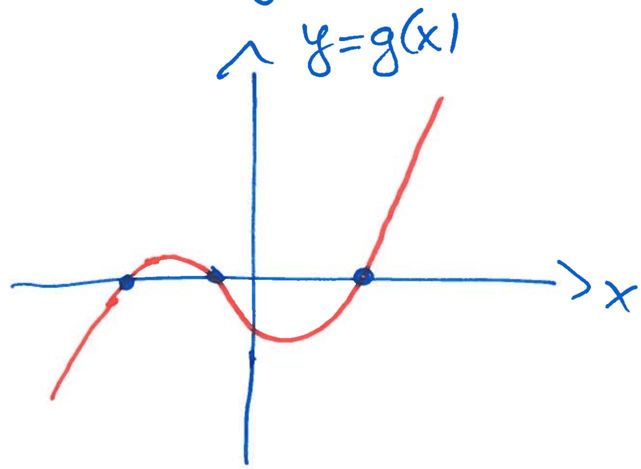
$f(x) = \sqrt{x}$



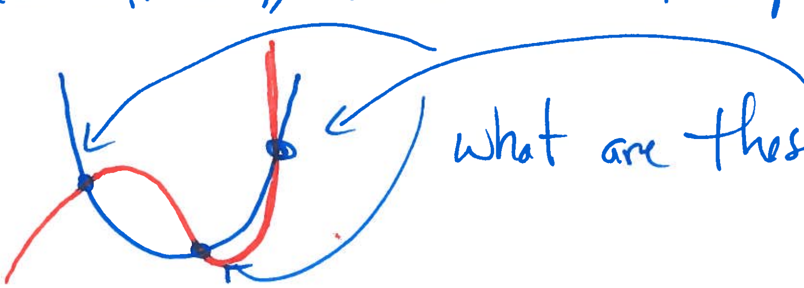
Where Graphs Meet

Consider the functions $f(x) = x^2 - 1$, $g(x) = x^3 - 2x - 1$.





For what values of x are $g(x)$ and $f(x)$ the same? Geometrically, this asks the question



what are these points?

I.e., where do these curves intersect?

Algebraically, we want to know the solutions
to

$$x^2-1 = x^3-2x-1$$

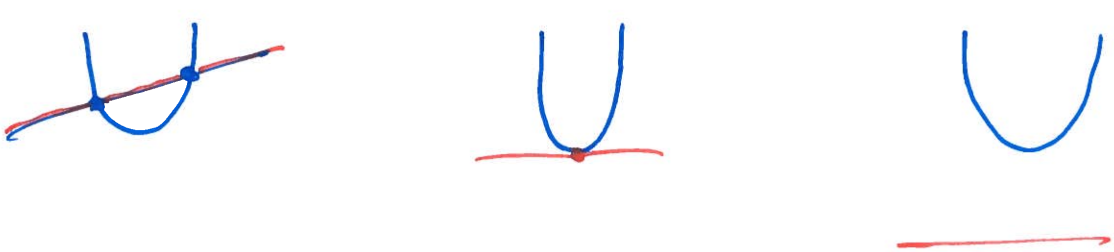
This is equivalent to solving

$$\begin{aligned} 0 &= x^3-2x-1-(x^2-1) \\ &= x^3-x^2-2x-1+1 \\ &= x^3-x^2-2x \quad \text{factor out an } x \\ &= x(x^2-x-2) \\ &= x(x-2)(x+1) \end{aligned}$$

So these intersect at $x=0$, $x=2$, and $x=-1$.

Intersecting a line and a parabola.

Geometrically, there are three possibilites



$f(x) = ax^2+bx+c$, $g(x) = mx+d$
(parabola) (line)

The points of intersection are the solutions to

$$f(x)-g(x) = ax^2+bx+c - (mx+d)$$
$$= ax^2 + (b-m)x + (c-d) = 0$$

Let $A = a$, $B = b-m$, $C = c-d$.

$$F(x) = f(x) - g(x) = Ax^2 + Bx + C = 0$$

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

Discriminant: $B^2 - 4AC$

2 Solutions : $B^2-4AC > 0$ 

1 Solution : $B^2 - 4AC = 0$ 

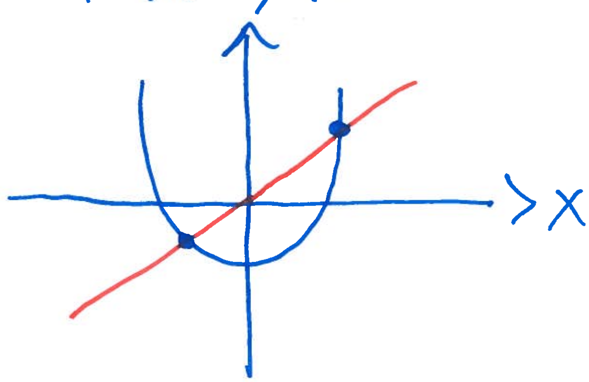
0 Solutions : $B^2 - 4AC < 0$ 

Real

Eg: $f(x) = x^2 - 2$, $g(x) = x$

$$f(x) - g(x) = (x^2-1) - x = x^2 - x - 2 = (x-2)(x+1)$$

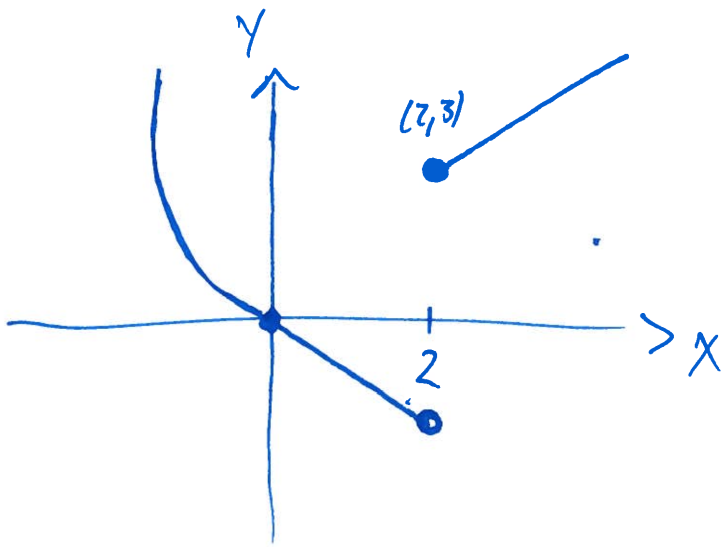
Intersect at $x=2$, $x=-1$.



Graphing Piecewise Functions

Recall: A piecewise function is a function that has different definitions on different parts of the domain.

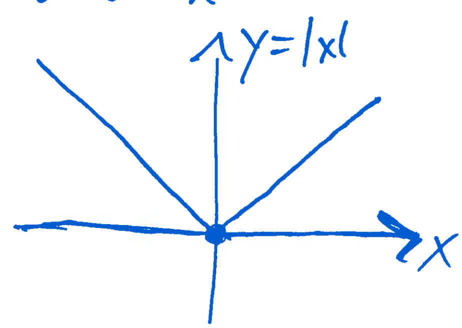
Eg: $$f(x) = \begin{cases} x+1 & x \geq 2 \\ -x & 0 \leq x < 2 \\ x^2 & 0 < x \end{cases}$$



Eg: The absolute value function, $|x|$.

$|x| = x$ when $x \geq 0$, $|x| = -x$ when $x < 0$

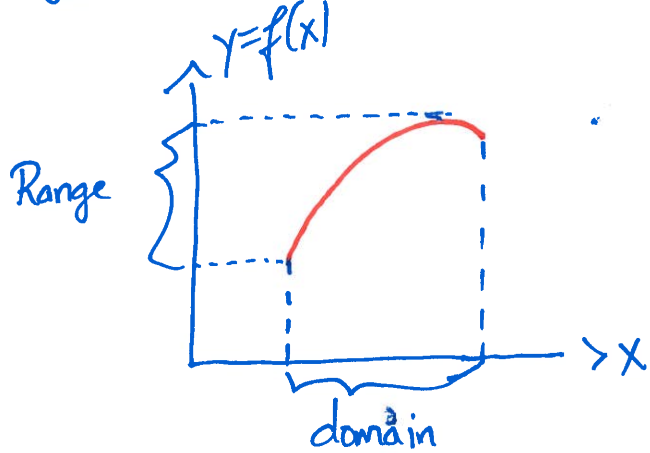
$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$



1.7:

Recall the domain of a function is the set of all possible input values.

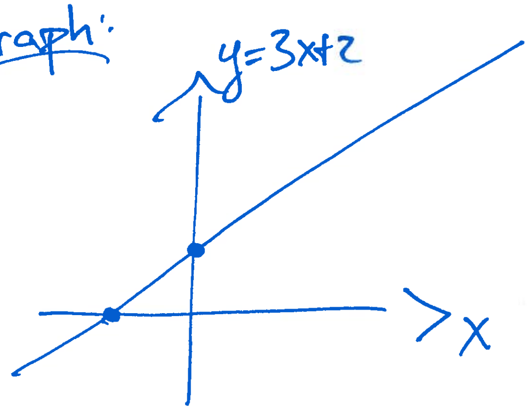
The range of a function is all the possible output values.



E.g: $f(x) = 3x + 2$ Domain $\mathbb{R}$

Range : $\mathbb{R}$

Graph:



Pick your favorite real number y, then you can find an x-value such that $3x+2=y$: it's given by

$$\frac{y-2}{3} = x$$

$$f\left(\frac{y-2}{3}\right) = 3\left(\frac{y-2}{3}\right) + 2 = y - 2 + 2 = y.$$