E.g: Annual Percentage Yield 10/26/17 ①

$\overset{P}{\overset{\shortparallel}{\$5000}}$ in a C.D., 5.5% yearly, compounded daily

Find the APY.

$$A(t) = P\left(1 + \frac{0.055}{365}\right)^{365t}$$

$$A(1) = P\underbrace{\left(1 + \frac{0.055}{365}\right)}_{\text{daily growth factor}}^{365} \quad \text{yearly growth factor}$$

$$a_{yearly} = \left(1 + \frac{0.055}{365}\right)^{365} \approx 1.0565$$

$$APY = r_{yearly} = a_{yearly} - 1 = 0.0565 = 5.65\%.$$

In general, for

$$A(t) = P(1+\tfrac{r}{n})^{nt}$$

the APY is

$$APY = \frac{A(1)}{P} - 1 = \frac{P(1+\frac{r}{n})^{n}}{P} - 1 = (1+\tfrac{r}{n})^{n} - 1$$

$\shortparallel$

yearly growth rate.

yearly growth factor.

3.3 Comparing Linear & Exponential Growth

Recall: Average rate of change of a function $f(x)$ from $a$ to $b$ is

$$\frac{f(b) - f(a)}{b-a} = \frac{f(a)-f(b)}{a-b}$$

The percentage rate of change is

$$\frac{f(x+1)-f(x)}{f(x)}$$

For an exponential function $f(x)=Ca^x$ this is the growth rate

$$\frac{f(x+1)-f(x)}{f(x)} = \frac{Ca^{x+1}-Ca^x}{Ca^x}$$

$$= \frac{Ca^x a - Ca^x}{Ca^x}$$

$$= \frac{\cancel{Ca^x}(a-1)}{\cancel{Ca^x}}$$

$$= a-1 = r.$$

Every exponential function has constant percentage rate of change and every function with constant percentage rate of change is exponential.

Every line has a constant average rate of change (slope) and every function with constant average rate of change is a line.

Eg:

| x | Y |
|---|---|
| 0 | 10,000 |
| 1 | 7,000 |
| 2 | 4900 |
| 3 | 3,430 |
| 4 | 2,401 |

a) Find the "first differences"

b) Find the % rate of change (from x to x+1) for all x.

Does this admit an exponential model?

| $x$ | $y$ | first diff. | % rate of change |
|---|---|---|---|
| 0 | 10000 | — | — |
| 1 | 7000 | $-3000 = 7000 - 1000$ | $\frac{-3000}{10000} = -30\%$ |
| 2 | 4900 | $-2100 = 4900 - 7000$ | $\frac{-2100}{7000} = -30\%$ |
| 3 | 3430 | $3430 - 4900 = -1470$ | $\frac{-1470}{4900} = -30\%$ |
| 4 | 2401 | $2401 - 3430 = -1029$ | $\frac{-1029}{3430} = -30\%$ |

Yes, an exponential decay model fits the data. We have

$r = -.3, \quad a = 1 + r = 1 + (-.3) = .7$

$$f(x) = 10{,}000(.7)^x$$

Eg.: 10,000 people in Newburgh. Two planners, predict pop. in 5 years.

Planner A: estimates growth by 500 people/year
Planner B: estimates growth by 5%/year.

$P_A(t) = 10000 + 500t$

$P_B(t) = 10000(1.05)^t \qquad (r = 0.5,\ a = 1 + r = 1.05)$

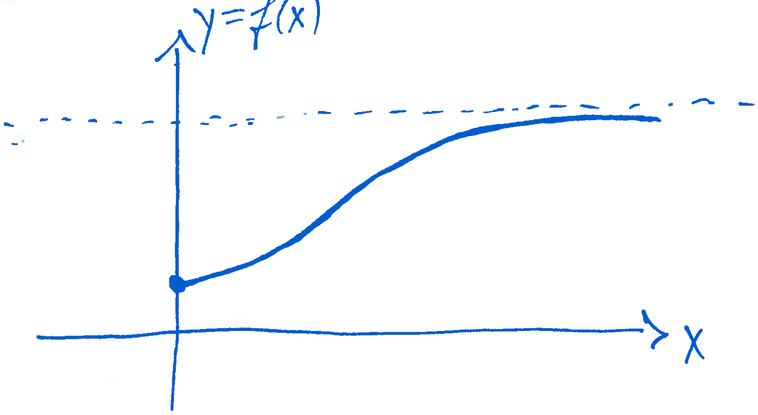
Eg.: Logistic Growth

A logistic growth model is a function of the form

$$f(x) = \frac{c}{1 + ba^{-x}} \qquad 1 < a,\ 0 < b$$

and models growth under limited resources.

The variable $x$ is the number of time periods,
The constant $C$ is called the carrying capacity, the maximum population the resources can support




$y = C$ ← horizontal asymptote

$$f(x) = \frac{C}{1+ba^{-x}}$$

$a^{-x} = \frac{1}{a^x}$, $1 < a$; as $x$ gets larger, $a^x$ gets very large (very fast for $a$ much larger than 1)

e.g: $a = 10$

$$a^0 = 1,\ a^1 = 10,\ a^2 = 100,\ a^3 = 1000,\ a^4 = 10{,}000,\ \dots$$

So $a^{-x}$ becomes small — tending towards zero!

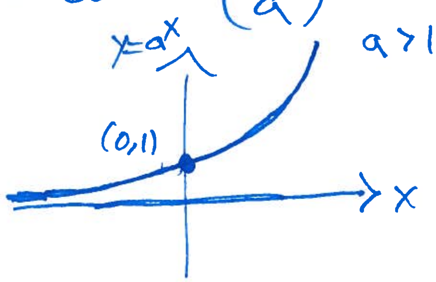
e.g: $a = 10$

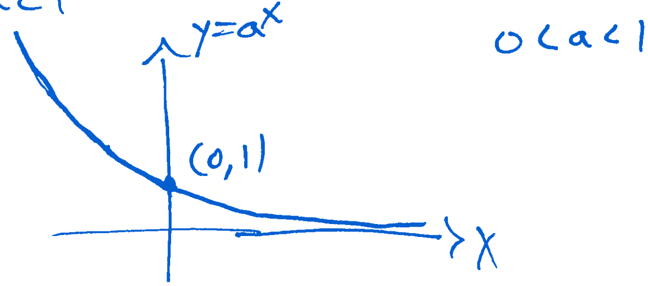
$$a^0 = 1,\ a^{-1} = .1,\ a^{-2} = .01,\ a^{-3} = .001,\ a^{-4} = .0001,\ \dots$$

## ~~E.g.~~ 3.4: Graphs of Exponential Functions

$f(x) = a^x$, $0 < a$, $a \neq 1$; domain is $\mathbb{R}$

$f(x) = a^{-x} = \left(\frac{1}{a}\right)^x \leftrightarrow 0 < a < 1$



Rmk: Exponential models (as used by the book) have thus far restricted the domain to $[0, \infty)$. Exponential functions have domain $\mathbb{R} = (-\infty, \infty)$.

Rmk: for an exponential function $f(x) = Ca^x$, the graph is the same shape as above, but passes through $(0, C)$.