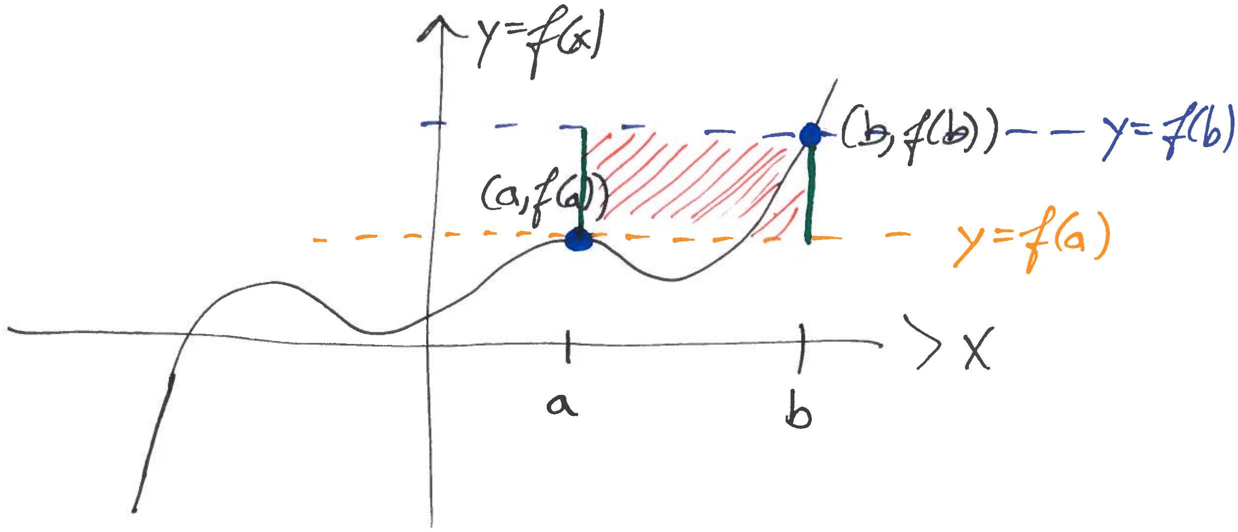
Evaluating Function - Net Change

Consider some function, f, the net change in f from a to b (where $a \le b$) is given by

$$f(b) - f(a).$$

Pictorially, the net change in the function f ~~f~~ from a to b is the length of the green line in the picture

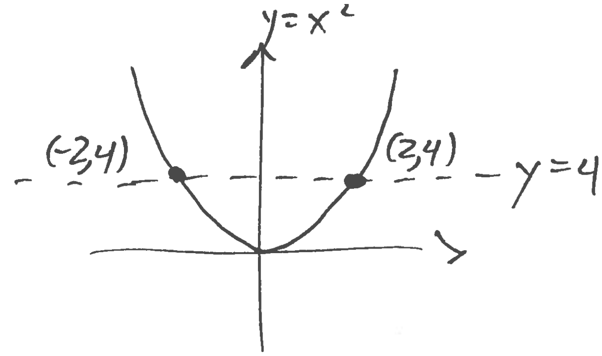


This is just saying that the net change is the height of the rectangle shaded in red.

Eg: What is the net change in $f(x) = x^2$ from -2 to 2?

$$f(2) - f(-2) = 2^2 - (-2)^2$$
$$= 4 - 4 = 0$$

This says something about symmetry.



Eg: net change of x^3-2x+1 from 0 to 3.

$$3^3-2(3)+1-(0^3-2(0)+1)=27-6+1-1$$
$$=21.$$

The Domain of a Function

The <u>domain</u> of a function, f, is the set of all possible input values.

Any polynomial will always have domain $\mathbb{R}$ (all real numbers).

$$p(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0, \text{ n an integer.}$$

<u>Power functions</u>

$$f(x)=x^n; \text{ let n be negative}$$

In this case,

$$x^n=\frac{1}{x^{-n}}$$

and the domain is just

$$\{x\in\mathbb{R} \mid x\neq 0\}$$

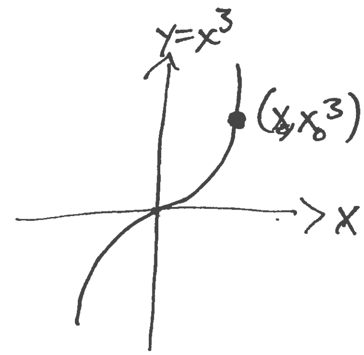
If the exponent is a rational, m/n,

$$x^{m/n} = (x^m)^{1/n} = (x^{1/n})^m$$

So the only thing we need to be careful about here is $x^{1/n}$.

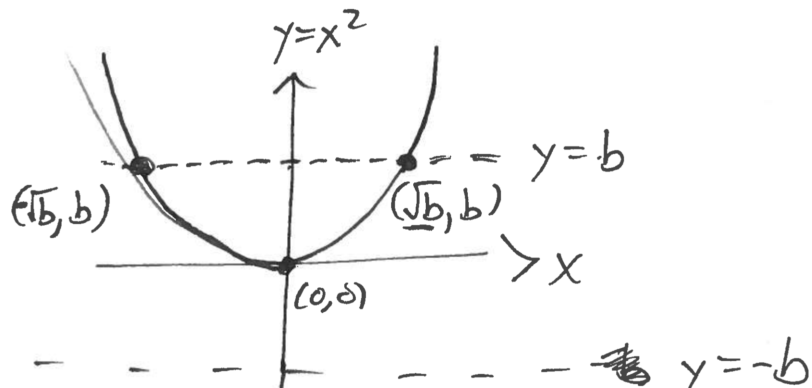
If n is odd, there's nothing to worry about: the domain is $\mathbb{R}$.

Eg: $\sqrt[3]{x} = x^{1/3}$, can be evaluated at every real number.



If n is an even number, this is not necessarily the case.

Eg:



For any ~~power~~ even n, $x^{1/n}$ has domain

$$\{x \in \mathbb{R} \mid x \geq 0\}$$

Rational Functions

For $p(x)$, $q(x)$ polynomials, the function

$$\frac{p(x)}{q(x)}$$

has domain

$$\{x \in \mathbb{R} \mid q(x) \neq 0\}.$$

E.g: $\dfrac{1}{x+2}$, find the domain

Find out where $x+2$ is zero, and exclude that point.

Solve $x+2=0$ for x: $x=-2$

Domain of $\frac{1}{x+2}$ is

$$\{x \in \mathbb{R} \mid x \neq -2\}.$$

E.g: $\dfrac{x^{50}+x^9+x^2-7000}{x^2+2x+1}$, find the domain.

Domain is all x such that $x^2+2x+1 \neq 0$.

Solve $x^2+2x+1=0$

$$x^2+2x+1=(x+1)(x+1)=(x+1)^2=0$$

only solution is $x=-1$

Domain is $\{x \mid x \neq -1\}$.

E.g: Ratio of two (not necessarily polynomial) functions.

$$\frac{\sqrt{x-2}}{x^2+x+1}, \text{ what is the domain?}$$

Find:
Domain of $\sqrt{(x-2)}$
Where $x^2+x+1=0$.

To find the domain of $\sqrt{x-2}$, we just need to find out where $0 \le x-2$; $2 \le x$

The roots of x^2+x+1 (i.e. the solutions to $x^2+x+1=0$) are given by

$$x = \frac{-1 \pm \sqrt{1^2-4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

Those are not real numbers, so the domain of

$$\frac{\sqrt{x-2}}{x^2+x+1}$$

is ~~x~~ $\{x \mid 2 \le x\}$

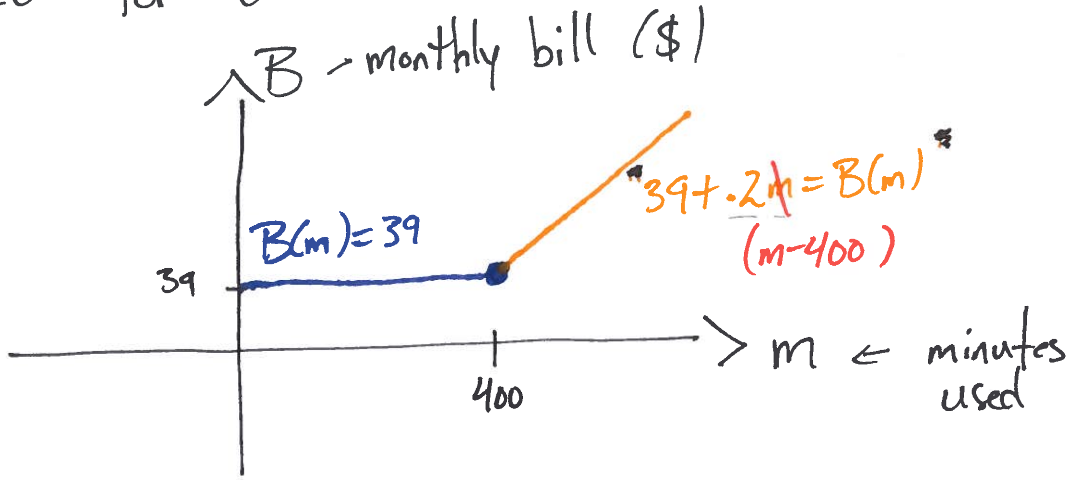
Piecewise Defined Functions

A piecewise function is a function that has multiple rules.

E.g.: A cell phone plan.

$39/mo for 400 minutes

$.20 for each additional minute.



$$B(m) = \begin{cases} 39 & 0 \le m \le 400, \\ 39 + .2(m-400) & 400 < m. \end{cases}$$