Which Equations Represent Functions? 9/19/17 1

An equation involving two variables can be thought of as a set of pairs

$$\{(x,y) \mid \text{"x and y make the equation true"}\}$$

E.g.: $y = x^2 - 1$ can be thought of as the set of all pairs of numbers, (x,y), such that

$$y = x^2 - 1.$$

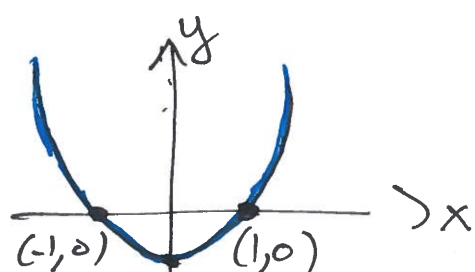
Set notation

$$\{(x,y) \mid y = x^2 - 1\} = \{(x, x^2-1) \mid x \in \mathbb{R}\}$$

Eg: $x = 4$, $y = x^2 - 1 = 4^2 - 1 = 16 - 1 = 15.$

$$(4,15) \in \{(x,y) \mid y = x^2 - 1\}$$

This is the graph of the function $y = x^2 - 1$



Equations that represent functions:

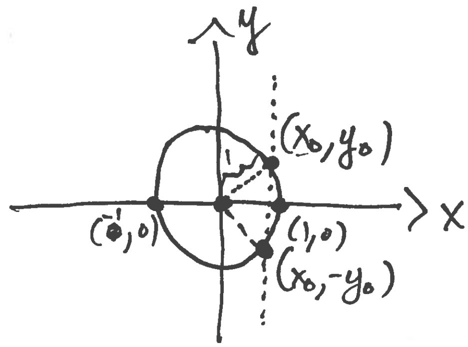
An equation involving variables x and y defines y as a function of x if and only if whenever given a value for x, there exists only one value of y such that the pair

(x,y) is an element of the set

$$\{(x,y) \mid \text{"x and y satisfy the equation"}\}$$

E.g: Consider the equation

$$x^2+y^2=1 \quad \text{(this is a circle of radius 1 centered at the origin (0,0))}$$



$(-1 \le x \le 1)$

This graph is just the set of pairs of numbers

$$\{(x,y) \mid x^2+y^2=1\}$$

If we know that (x_0,y_0) satisfies

$$x_0^2+y_0^2=1$$

then also

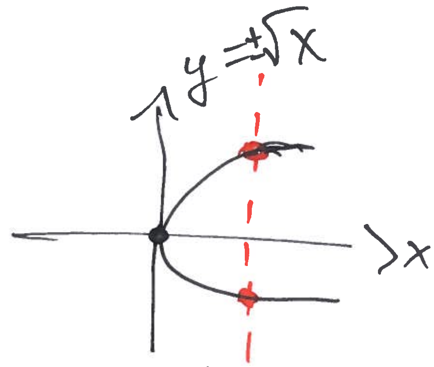
$$x_0^2+(-y_0)^2 = x_0^2+y_0^2=1 \;\&$$

so long as $x_0 \neq \pm 1$, $y_0 \neq -y_0$, so $(x_0,-y_0)$ is also an element of

$$\{(x,y) \mid x^2+y^2=1\}.$$

The vertical line test says that if any vertical line intersects the graph of an equation in more than one point, then that equation does not define y as a function of x. ③

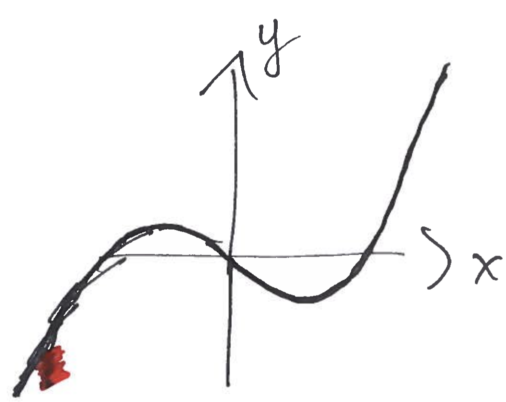
E.g:



The associated equation does not define y as a function of x.

Conversely, if any vertical line intersects the graph in at most one point, then it is the graph of a function.

E.g:



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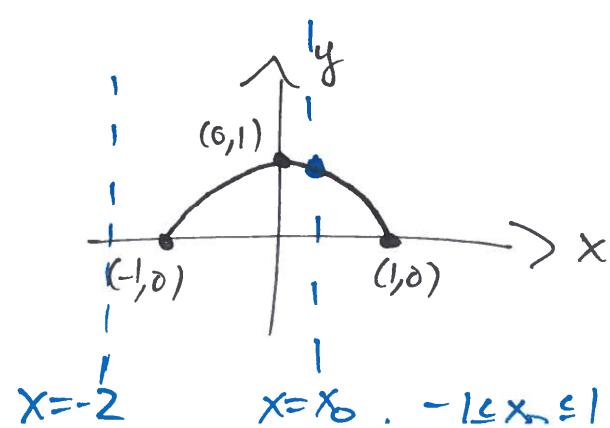

E.g: $5z + 2w^2 = 8$

a) is z a function of w?

b) is w a function of z?

a) Solve for z in terms of w: subtract $2w^2$ from both sides:

$$5z = 8 - 2w^2$$

Divide both sides by 5:

$$z = \frac{8 - 2w^2}{5}$$

Yes, z is a function of w.

b) Solve for w in terms of z.

Subtract $5z$ from both sides.

$$2w^2 = 8 - 5z$$

Divide both sides by 2:

$$w^2 = \frac{8 - 5z}{2}$$

Take the square root of both sides:

$$w = \pm\sqrt{\frac{8 - 5z}{2}}$$

~~This is~~ w is not a function of z because of the choice of sign for w.

Eg: $x^2+y^2=1$

a) is x a function of y?

b) is y a function of x?

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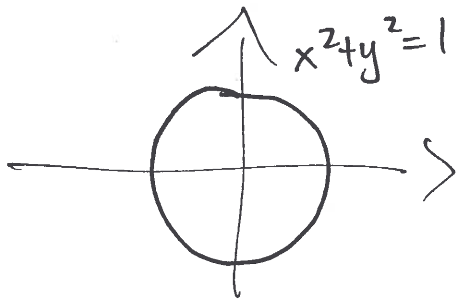
b) Solve for y in terms of x: sub. x^2 from both sides:

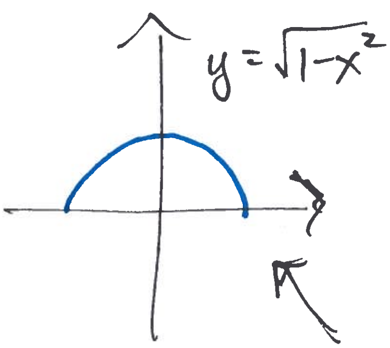
$$y^2 = 1-x^2$$

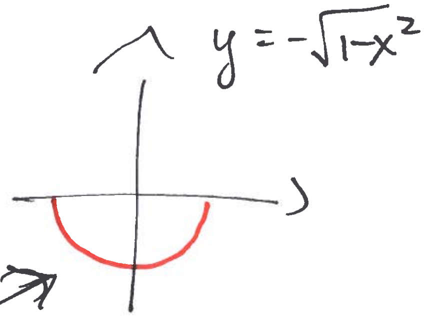
take sq. roots:

$$y = \pm\sqrt{1-x^2}$$

a graph of a not function







graphs of functions

1.5: Function Notation: The Concept of Function as a Rule

Usually one gives a function a name. Usually that name is f. (other names may include, e.g., g or h)

There is a variable, x, which represents the input to the function, f. This is represented by

$f(x)$ – output of f on x

"image of x under f"

Sometimes one writes

$$y = f(x).$$

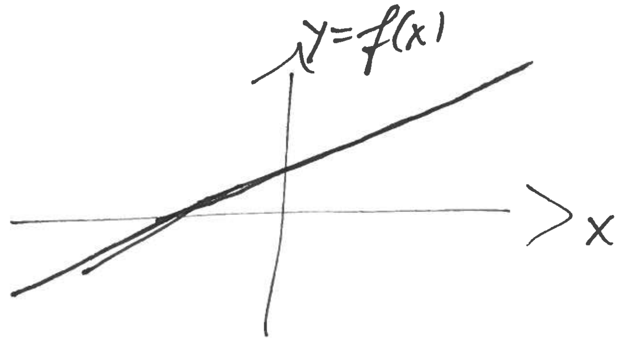
Eg: $y = 3x+4$ – this is a function. One could name it f and write

$$f(x) = 3x+4.$$

$\{(x, 3x+4) \mid x \in \mathbb{R}\}$

$\quad\quad\; ||$

$\{(x, f(x)) \mid x \in \mathbb{R}\}$



Where one could say "evaluate $3x+4$ at $x=4$,"[1] one can just as well say "$f(4)$"[2]

(1) $3(4)+4 = 12+4 = 16.$

(2) $f(4) = 3(4)+4 = 12+4 = 16.$

*read as "f of 4" or as (1)

Eg: Express the given rule in function notation: 7

a) Multiply by 2, then add 5

b) Add three, then square.

a) Let's name the function g and the variable n.

$$g(n) = 2n+5$$

b) f, the name of the function, x the variable:

$$f(x) = (x+3)^2$$