For data ~~set~~ given by some table, e.g.

x (chairs)	C (cost \$)
0	80
1	92
2	104
3	116
4	128

$\longleftrightarrow$ linear model

$$C = 80 + 12x$$

the data is said to be evenly spaced if the inputs (in the example above, the chairs) are 1 unit apart. For data with evenly spaced inputs, the first differences are the differences in successive outputs.

To say data of this kind admits a linear model is equivalent to requiring that the first differences are constant.

For any linear model

$$y = A + Bx$$

if you look at the y-values associated to x and $x+1$, they are

$$A + Bx \quad \text{and} \quad A + B(x+1)$$

the difference between these two numbers is

$$A + B(x+1) - (A + Bx) = A + Bx + B - A - Bx$$
$$= B$$

Consider the table

x	y
0	1
1	2
2	4
3	8
4	16

Does this admit a linear model?

$$2-1=1$$
$$4-2=2$$

The answer is no: the first differences are not all the same.

Eg.:

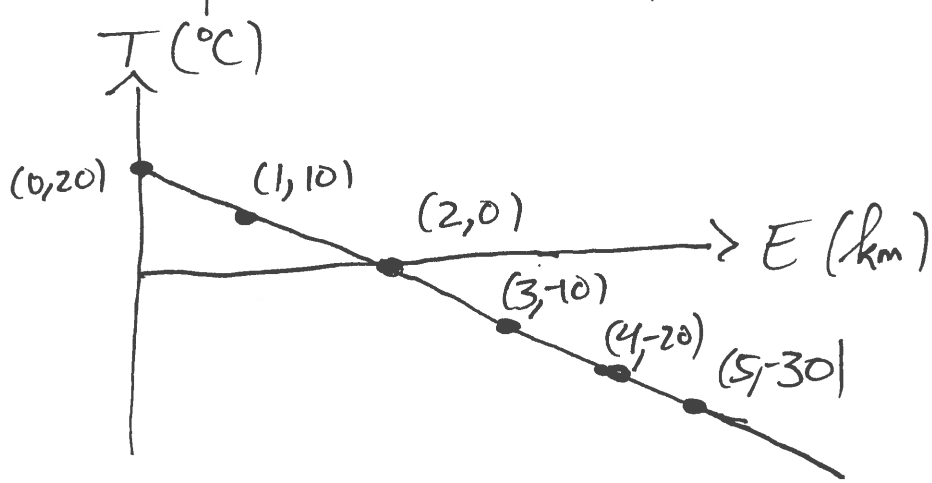
Elevation (km)	Temp (°C)
0	20 ← initial value (A)
1	10
2	0
3	−10
4	−20
5	−30

The first differences are all the same, −10. ← B

linear model

$$T = 20 - 10E$$

Important that E is the elevation in kilometers



Eg.:

Depth (ft)	Pressure (lb/in^2)
0	14.7
10	19.2
20	23.7
30	28.2
40	32.7
50	37.2

The first differences are all 4.5

$19.2 - 14.7 = 4.5$

$23.7 - 19.2 = 4.5$

$\vdots$

$P = A + BD$

P - pressure in psi

D - depth in ~~10s of~~ feet

Normalize to feet by taking

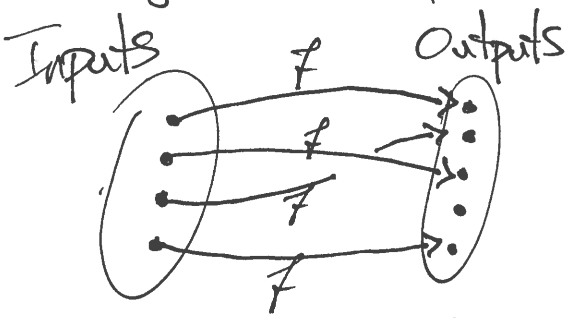
$B = \frac{4.5}{10}$

$A = 14.7$

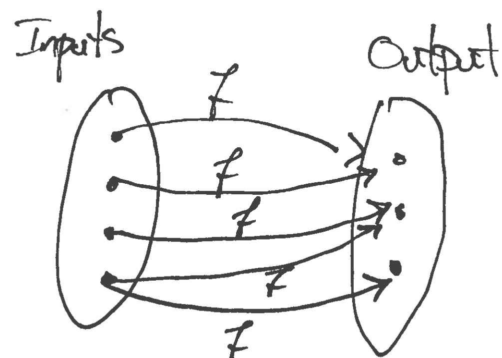
$$P = 14.7 + \frac{4.5}{10} D$$
$$= 14.7 + .45 D$$

1.4 Functions: Describing Change.

Def^n: A function is a relation in which each input gives exactly one output.



"Picture" of a function



"Picture" of a non-function

Take the set

$$D = \{1, 2, 3, 4\}$$

and the set

$$R = \{20, 30, 42, \pi\}$$

Define a function, f, that assigns 1 to 42, 2 to π, 3 to 42, 4 to 30. One can represent such a function by a table

D	R
1	42
2	π
3	42
4	30

Usually one writes such a table with inputs on the left and outputs on the right.

Eg: Table of ages of women in a nbhd and number of children

Age	number
31	3
32	0
24	1
35	1
31	2
22	0

not a function

- The # of kids is not a function of the age of the women
- The age of the women is not a function of the number of kids.

There are two types of variables: independent and dependent.

1. A variable y is a function of a variable x if each value x corresponds to exactly one value of y.

   (If you first choose a value for x, there exists only one ~~choice~~ value of y corresponding to that choice).

2. In this case x is the <u>independent variable</u>, y is the <u>dependent variable</u>

E.g.: $y = A + Bx$ for any choice of $A, B \in \mathbb{R}$

this is a function.

E.g.:

x	y
1	22
2	22
3	28
4	31
5	34
6	37

dependent

independent

y is a function of x.

However, x is <u>not</u> a function of y because one would have to make a choice of assigning 22 to 1 or to 2.

Net Change in the Dependent Variable

E.g.:

year (x)	y (dollars)
'96	1.32
'97	1.33
98	1.16
99	1.36
2000	1.66
'01	1.64
'02	1.51
'03	1.83

y - avg annual gas price in California

x - year

y is a function of x

The net change in avg. annual gas price from 1996 to 1998

'04	2.16
'05	2.17
'06	2.81

$1.16 - 1.32 = -.16$

Net change is a 16 cent decrease.

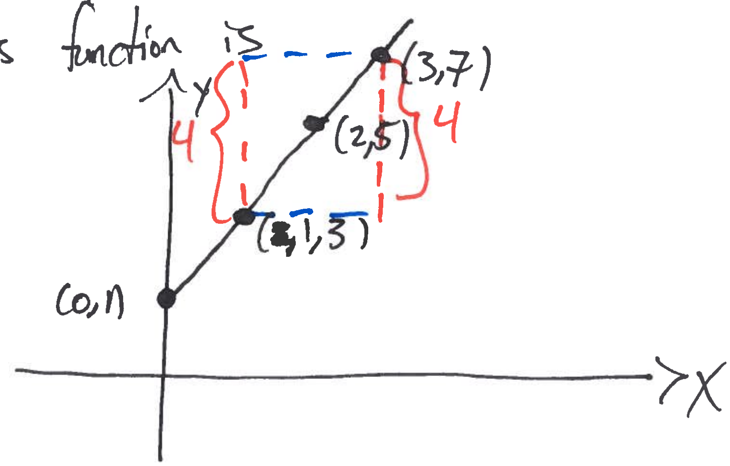
If y is a function of x, the net change in y between two values $x_0 < x_1$ is the difference in the y-values, y_0 and y_1, associated to x_0 and x_1, respectively

$$y_1 - y_0.$$

E.g.: Let's say we have the linear model

$$y = 2x + 1$$

The graph of this function is



The net change from 1 to 3 is

$$7 - 3 = 4$$

The net change is the length of the red line