E.g.- Solve

$$2 + \frac{5}{x-4} = \frac{x+1}{x-4}$$

for x.

Multiply both sides by x-4

$$(x-4)\left(2+\frac{5}{x-4}\right) = \left(\frac{x+1}{x-4}\right)(x-4)$$

$$(x-4)2 + 5 = x+1$$

$$2x-8+5 = 2x-3 = x+1$$

$$\Rightarrow 2x-3-x = x+1-x$$

$$\Rightarrow x-3 = 1$$

$$\Rightarrow x-3+3 = 1+3$$

$$\Rightarrow x = 4$$

Extraneous solution. This is because ~~x~~ 4-4=0, and we cannot divide by zero. There are no real solutions to this equation.

C.3 Solving Inequalities

An inequality is just two expressions separated by

$<, \leq, >, \geq$

Eg.: $4x+7 \leq 19$

Operations on Inequalities

1. Add/subtract the same quantity to/from both sides of the inequality.
2. Multiply/divide by the same positive quantity on both sides
3. Multiply/divide by the same negative quantity on both sides and flip the symbol.

E.g.: Solve $3x < 9x+4$. (linear inequality)
Subtract $9x$ from both sides

$$-6x < 4$$

Divide both sides by $-6$ and flip the symbol

$$x > \frac{4}{-6} = \frac{-2}{3}$$

Graphically:


E.g.: $x^2 - 2x - 8 \le 0$
Start by solving $x^2 - 2x - 8 = 0$
Factor $x^2 - 2x - 8 = (x-4)(x+2)$; this says

$$x^2 - 2x - 8 = 0$$

if either $x = 4$ or $x = -2$.


The solutions to $x^2 - 2x - 8 = 0$ partition the number line into three intervals

$(-\infty,-2),\ (-2,4),\ (4,\infty)$

Test one number from each interval and test to see if it satisfies the strict inequality

$$x^2-2x-8<0$$

If it does, so does every other number in that interval.

We checked

$$0^2-2(0)-8=-8<0$$

so every number in $(-2,4)$ also satisfies this inequality.

We check

$$(-3)^2-2(-3)-8=9+6-8=1+6=7\not<0$$

so no other number in $(-\infty,-2)$ satisfies this inequality.

We check

$$(5)^2-2(5)-8=25-10-8$$
$$=15-8=7\not<0$$

so no other number in $(4,\infty)$ satisfies this equation.

The solutions to

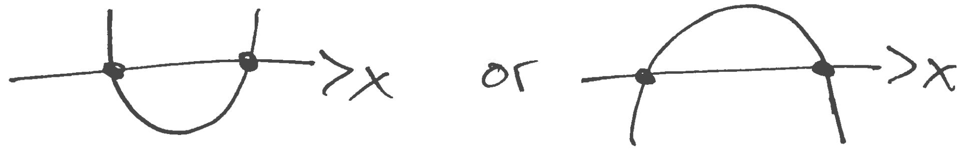
$$x^2-2x-8<0$$

are the numbers in the interval

$$[-2,4]$$

Hand wavy justification: every degree 2 polynomial has a graph that looks like

Having 2 solutions says the x-axis passes through the graph in two points




E.g.: Solve the inequality

$$x^2 > 2x+3.$$

Put everything on the left: subtract $2x+3$ from both sides

$$x^2-2x-3>0$$

Solve $x^2-2x-3=0$: factor

$$x^2-2x-3 = (x-3)(x+1) = 0$$

Two solutions are $x=3$, $x=-1$; they are not solutions to the inequality.
to the equation.


Check 0: $0^2-2(0)-3 \not> 0$

Check -2: $(-2-3)(-2+1)=(-5)(-1)>0$

Check 4: $(4-3)(4+1)=1(5)>0$

Solutions to $x^2-2x-3>0$ are given by the union of intervals

$$(-\infty,-1)\cup(3,\infty) \quad (\text{any } x<-1 \text{ or any } 3<x).$$

# Ch 1.3: Equations: Describing Relationships in Data

## Making a Linear Model from Data

A model is a mathematical representation of some phenomenon.
A linear model is an equation of the form

$$y = A + Bx, \quad A, B \in \mathbb{R}$$

The value $A$ is called the initial value; this is the value of $y$ when the variable $x = 0$.
The value $B$ is the amount by which $y$ changes for each unit increase in $x$.

E.g.: Table

| $x$ chairs | $C$ dollars ← cost |
|---|---|
| 0 | 80 ← initial cost |
| 1 | 92 |
| 2 | 104 |
| 3 | 116 |
| 4 | 128 |

The chair maker wants to represent the cost, $C$, in terms of the number of chairs produced, $x$, as a linear model.

Each time a chair is produced, the cost increases by \$12

$$C = 80 + 12x$$

This is a linear model by definition, and it fits the data in the table in the sense that

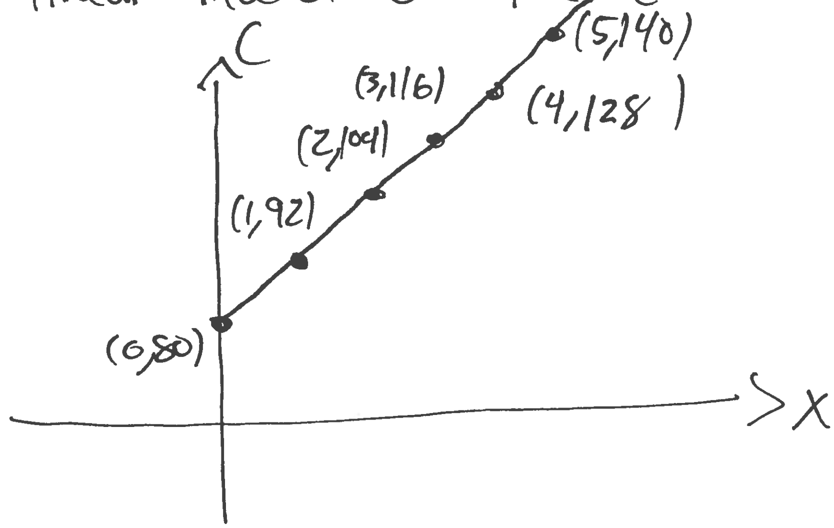
$$80 = 80 + 12(0)$$

$$92 = 80 + 12(1)$$

$$\vdots$$

$$128 = 80 + 12(4).$$

Plot the linear model on the cartesian plane




Linear models are called "linear" because they are lines; that is to say that the graph of a linear model is a line.