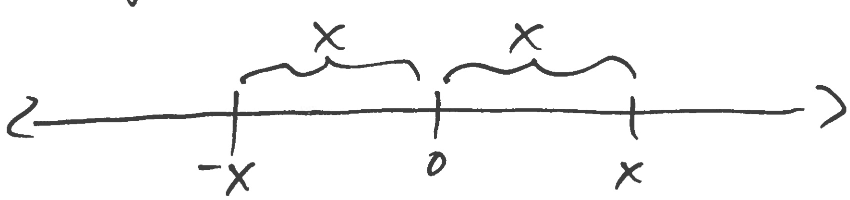
A.2: The Number Line & Intervals

The real numbers can be represented as a line with a marked point, called the origin, which represents the number 0.

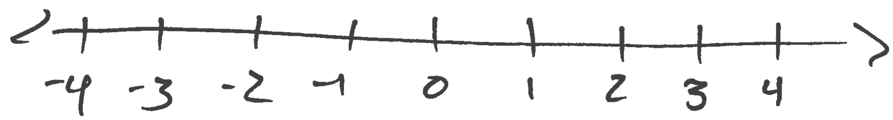
Every positive number, x, lies x units to the right of the origin, and every negative number, $-x$, lies ~~to~~ x units to the left of the origin.



The points on the line represent a real number and we call that value its <u>coordinate</u>.

One calls this either the <u>coordinate line</u>, the <u>real number line</u>, or the <u>real line</u>.

Eg:



<u>Ordering</u>

For any two real numbers, a and b, we say

(i) $a < b$ if $b-a$ is positive ("a is less than b")

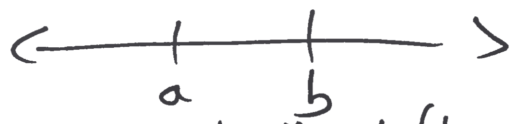
(ii) $b < a$ if $a-b$ is positive ("b is less than a")

(iii) $b = a$ if $b-a = 0 = a-b$ ("a is equal to b").

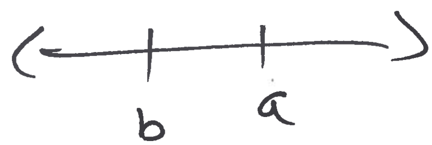
This relation gives the real numbers ~~and~~ an <u>ordering</u>, which means exactly one of these three conditions holds for any two real numbers.

Geometrically,

(i) means a lies to the left of b on the real line



(ii) means b lies to the left of a



(iii) means a and b are two different names for the same number.

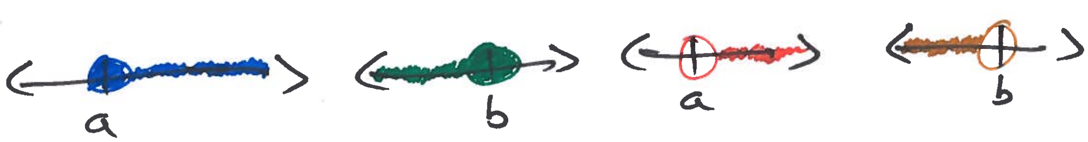
The symbol "$\leq$" means "less than or equal to." To say $a \leq b$ is to say that a is no larger than b.

<u>Graphing</u> <u>Inequalities</u>

Usually one reserves x for a variable, and letters like a, b, c, etc. will usually denote fixed real numbers.

There are four flavors of inequalities:

$$a \leq x \qquad x \leq b \qquad a < x \qquad x < b$$



<u>Sets</u>

A <u>set</u> is a collection of objects called <u>elements</u>.

We denote member in a set, A, by

$a \in A$ "a is an element of A" or "a is in A"

E.g: $A = \{1, 2, 3, 4\}$

This is the set with elements 1, 2, 3, 4 and its name is A.

We could write

$$1 \in A \quad \text{or} \quad 2 \in A \quad \text{or} \quad 3 \in A \quad \text{or} \quad 4 \in A.$$

When the set has too many elements to write down explicitly, one uses set-builder notation:

A = { x | <condition> }

↖ "such that"

"A is the set of objects, x, satisfying <condition>."

E.g: $\{x \mid x \text{ is an integer greater than zero and less than five}\}$

$= \{1, 2, 3, 4\}$

Alternatively: $\{x \mid 0 < x < 5, x \text{ an integer}\}$

Special Sets

$\mathbb{N}$ — the set of positive integers $(1, 2, 3, 4, \ldots)$ called the natural numbers.

$\mathbb{Z}$ — the set of integers $(\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots)$

$\mathbb{Q}$ — the set of rational numbers (a/b, $a \in \mathbb{Z}$, $b \in \mathbb{Z}$, e.g. $3/2$, $4/3$, etc)

$\mathbb{R}$ — the set of all real numbers.

Operations on Sets

We're concerned with union and intersection.

For two sets A and B, the union of A and B is the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

The intersection of A and B is the set

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

E.g.: $A = \{1, 2, 3, 4\}$, $B = \{4, 5, 6, 7\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A \cap B = \{4\}$$

Two sets don't necessarily have to have a common element.

E.g.: $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$

We define the empty set, $\emptyset$, which is the set without any elements. So

$$A \cap B = \emptyset$$

In this case we say A and B are disjoint.

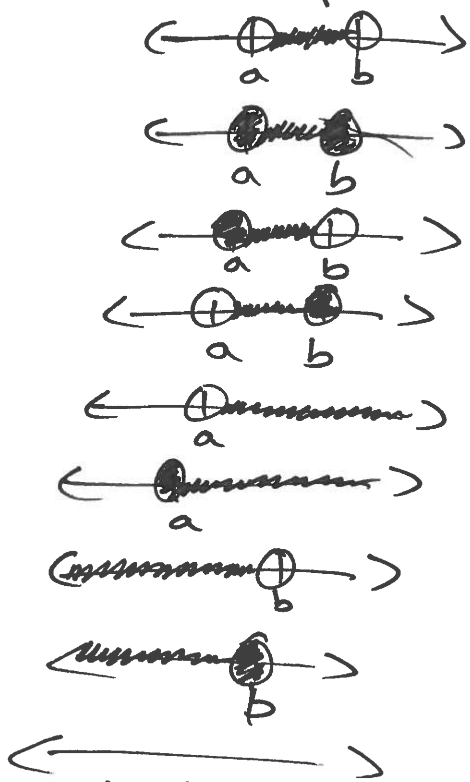
Intervals

An interval is an unbroken piece of the number line.

The types of intervals are

Interval	Set Notation
(a,b)	$\{x\in\mathbb{R} \mid a<x<b\}$
$[a,b]$	$\{x\in\mathbb{R} \mid a\le x\le b\}$
$[a,b)$	$\{x\in\mathbb{R} \mid a\le x<b\}$
$(a,b]$	$\{x\in\mathbb{R} \mid a<x\le b\}$
(a,∞)	$\{x\in\mathbb{R} \mid a<x\}$
$[a,\infty)$	$\{x\in\mathbb{R} \mid a\le x\}$
$(-\infty,b)$	$\{x\in\mathbb{R} \mid x<b\}$
$(-\infty,b]$	$\{x\in\mathbb{R} \mid x\le b\}$
$(-\infty,\infty)$	$\mathbb{R}$

~~Ex~~ Graphs



the whole real line.