MATH 111
EXAM 02 SOLUTIONS

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1. Definitions

1 (4 Points). (a) State the Point-Slope form of a line passing through the point (x_0, y_0) with slope m.

Solution.

$$y - y_0 = m(x - x_0)$$

(b) State the Slope-Intercept form of a line with slope m and y-intercept b.

Solution.

$$y = mx + b$$

2 (6 Points). Let $f(x)$ be a function. State the average rate of change of f between $x = a$ and $x = b$.

Solution.

$$\frac{f(b) - f(a)}{b - a} = \frac{f(a) - f(b)}{a - b}$$

3 (5 Points). Let $f(x)$ be an exponential function and let a be the growth/decay factor. Express the growth/decay rate, r, in terms of a.

Solution.

$$r = a - 1$$

4 (3 Points). (a) State the general form of an exponential function.

Solution.

$$Ca^x$$

(b) When does such a function model exponential growth?

Solution. When $1 < a$.

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(c) When does such a function model exponential decay?

Solution. When $0 < a < 1$.

5 (2 Points)**.** Consider the two lines $f(x) = m_1x + b_2$ an $g(x) = m_2x + b_2$.

(a) When are f and g parallel?

Solution. When $m_1 = m_2$.

(b) When are f and g perpendicular?

Solution. When any of the following three equivalent conditions occur

- $m_1m_2 = -1$,
- $m_1 = \frac{-1}{m_2}$, or
- $m_2 = \frac{-1}{m_1}$.

2. Problems

6 (16 Points)**.** In the following problems, use the given information to find the equation of the line in slope-intercept form.

(a) The line passing through the points $(-2, 3)$ and $(5, -18)$.

Solution. The slope of the line between these points is

$$\begin{aligned} m &= \frac{3-(-18)}{-2-5} \\ &= \frac{3+18}{-7} \\ &= -\frac{21}{7} \\ &= -3. \end{aligned}$$

The point-slope form of this line is

$$y - 3 = -3(x - (-2)) = -3(x + 2)$$

and the slope-intercept form is

$$y = -3x - 6 + 3 = -3x - 3$$

(b) The line passing through the point $(3, -2)$ and parallel to the line $2y - 6x = 8$.

Solution. We can put the given line into slope-intercept form by first diving both sides by 2 to get

$$y - 3x = 4$$

then adding $3x$ to both sides to get

$$y = 3x + 4.$$

Thus the slope of the parallel line is also 3. In point-slope form the desired line is

$$y - (-2) = 3(x - 3).$$

The slope-intercept form is

$$y = 3x - 9 - 2 = 3x - 11.$$

(c) The line passing through the origin (that is, the point $(0, 0)$) and perpendicular to the line $4y - x = 8$.

Solution. Adding x to both sides of the given equation and then dividing both sides by 4 we see that the slope-intercept form of the line is

$$y = \frac{x}{4} + 8,$$

so the slope of a perpendicular line is -4. Therefore the slope-intercept form of the desired line is

$$y = -4x.$$

7 (16 Points)**.** Consider the two lines $f(x) = x + 2$ and $g(x) = 3x + 4$. Find the point (that is, the (x, y) pair) where these two lines intersect.

Solution. To find the point of intersection we need only solve the equation

$$x + 2 = 3x + 4$$

for x. Subtracting x from both sides we get

$$2 = 2x + 4.$$

Subtracting 4 from both sides we get

$$-2 = 2x.$$

Finally, dividing both sides by 2 we get

$$x = -1.$$

The y-coordinate is given by

$$y = -1 + 2 = 1$$

so the point of intersection is $(-1, 1)$.

8 (16 Points)**.** Let $f(x) = x^2 - 2$.

(a) Compute the average rate of change for f between $x = 2$ and $x = 5$.

Solution. The average rate of change is

$$\begin{aligned} \frac{f(5) - f(2)}{5 - 2} &= \frac{(25 - 2) - (4 - 2)}{3} \\ &= \frac{25 - 2 - 4 + 2}{3} \\ &= \frac{25 - 4}{3} \\ &= \frac{21}{3} \\ &= 7. \end{aligned}$$

(b) Give the Point-Slope form of the line that passes through $(2, f(2))$ and $(5, f(5))$.

Solution. The slope of this line is 7, as computed above. The point-slope form of the line is

$$y - 2 = 7(x - 2).$$

(c) Give the Slope-Intercept form of the line that passes through $(2, f(2))$ and $(5, f(5))$.

Solution. The slope-intercept form of the line is

$$y = 7x - 14 + 2 = 7x - 12.$$

9 (16 Points)**.** Alice is hosting an event. She is renting a facility, which costs \$150, and providing refreshments, which cost \$7 per guest.

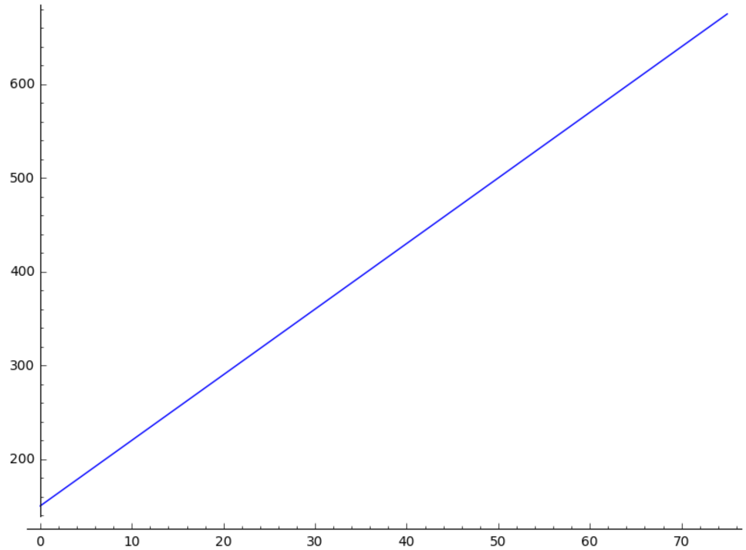
(a) Find a function, C, that models the total cost of the event if x people attend.

Solution.

$$C(x) = 7x + 150.$$

(b) Sketch a graph of C.

Solution. The function is a line starting from $(0, 150)$.



(c) Evaluate $C(10)$ and $C(15)$. What do these numbers represent?

Solution. The value

$$C(10) = 7(10) + 150 = 70 + 150 = 220$$

represents the cost if 10 people attend and the value

$$C(15) = 7(15) + 150 = 105 + 150 = 255$$

represents the cost if 15 people attend.

(d) If the total cost for the event was \$500, how many people attended?

Solution. To find the number of people that attended the party, solve

$$500 = C(x) = 7x + 150$$

for x. The solution is given by subtracting 150 from both sides of the equation then dividing both sides of the equation by 7, so

$$x = \frac{500 - 150}{7} = \frac{350}{7} = 50.$$

Therefore 50 people attended.

10 (16 Points)**.** A population of size 32 grows by 25% every day.

(a) Give the daily growth factor for this population.

Solution. We are given the daily growth rate

$$r = 25\% = \frac{25}{100} = \frac{25}{4(25)} = \frac{1}{4}$$

so the daily growth factor is given by

$$a = 1 + r = 1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}.$$

(b) Give an exponential model for the size of the population after t days.

Solution. The population is modeled by

$$P(t) = 32\left(\frac{5}{4}\right)^t.$$

(c) Determine the size of the population after 2 days.
[Hint: Express the growth factor as a fraction, rather than a decimal, and this will be very easy to compute.]

Solution. The population after 2 days is

$$P(2) = 32\left(\frac{5}{4}\right)^2 = 32\left(\frac{5^2}{4^2}\right) = 32\left(\frac{25}{16}\right) = 2(25) = 50.$$